

RECONFIGURABLE CONTROL DESIGN WITH INTEGRATION OF A REFERENCE GOVERNOR AND RELIABILITY INDICATORS

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A new approach to manage actuator redundancy in the presence of faults is proposed based on reliability indicators and a reference governor. The aim is to preserve the health of the actuators and the availability of the system both in the nominal behavior and in the presence of actuator faults. The use of reference governor control allocation is a solution to distribute the control efforts among a redundant set of actuators. In a degraded situation, a reconfigured control allocation strategy is proposed based on on-line re-estimation of the actuator reliability. A benefit of incorporating reliability indicators into over-actuated control system design is the smart management of the redundant actuators and improvement of the system safety. Moreover, when the fault is severe, an adaptation approach using the reference governor is proposed. The reference governor unit is a reference-offset governor based on a discrete-time predictive control strategy. The idea is to modify the reference according to the system constraints, which become stricter after the occurrence of an actuator fault. The proposed approach is illustrated with a flight control application.

Keywords: fault tolerant control systems, control allocation, reliability estimation, performance degradation, reference governor.

1. Introduction

In order to satisfy the increased economic demand for high plant availability and system safety, dependability is becoming an essential need in industrial automation. In this context and to satisfy these requirements, Fault-Tolerant Control (FTC) is introduced. The aim of FTC is to keep a plant available by the ability to achieve the objectives that have been assigned in the faulty behavior and accept reduced performance when critical faults occur (Blanke *et al.*, 2006; Noura *et al.*, 2009). In most safety critical systems, actuator redundancy is often used, e.g., in the three major control effectors in aircraft flight control (aileron, elevator and rudder). They are usually designed utilizing one control effector or actuator for each rational degree of freedom. However, due to the increased requirements on the reliability, maneuverability and survivability of modern and future aircraft, control effectors are no longer limited to these three conventional control effectors and many more control actuators have been introduced.

Moreover, several tools and approaches have been proposed to manage redundancy and to distribute the de-

sired control efforts among a set of actuators. A common approach is to use optimal control theory to shape the closed-loop dynamics and to distribute the desired control efforts in one step. Optimized methods like linear quadratic control (Kwakernaak and Sivan, 1972) and robust control (Zhou *et al.*, 1996) are readily available. An alternative strategy is to separate the regulation task from control distribution. Indeed, the control law specifies only the desired control efforts to be produced, and a separate *control allocation module* is introduced in the control loop to distribute the control among the actuators (Harkegard, 2003). This strategy is used in practical applications in aerospace control.

In the work of Durham (1993), control allocation is applied to several airplane flights. Bordignon (1996) illustrates this technique for an F-18 fighter with seven independent moments. A review of existing methods can be found in the work of Enns (1998). In the degraded functional, reconfigurable control allocation is employed. The advantage of this strategy is the ability to accommodate the control surface damages without modifying the

controller parameters. Various approaches to control re-allocation have been proposed for flight control systems. Alwi and Edwards (2008) propose an on-line control allocation with a sliding modes controller. Burken *et al.* (2001) reformulate the control allocation problem based on a quadratic programming problem. Reconfigurable control allocation based on pseudo-inverse methods is illustrated by Zhang *et al.* (2007). Recently, Johansen and Johansen (2008) as well as Casavola and Garone (2010) have proposed an adaptive control allocation approaches.

The main goal of these methods is to improve the safety and reliability of the system, which is rarely associated with an objective criterion that guides a design (Theilliol *et al.*, 2009). Some works have introduced reliability analysis for fault-tolerant control systems in order to take into account the health of the actuators in the reconfiguration strategy (Guenab *et al.*, 2006). Reconfigurability analysis was investigated for a reliable fault-tolerant control design by Khelassi *et al.* (2009). In this context, a reconfigurable control allocation design with integration of the reference governor is proposed in this paper based on actuator reliability. The aim is to smartly manage the redundant actuators in order to satisfy the performance requirements and improve the probability of the success of the mission.

Modification of the control law following the re-allocation step moves the faulty operating point to a degraded one. Under this operating point, some problems may occur due to physical limitations inherent to the system. In particular, FTC must take into consideration the input/state constraints which are dictated by actuator saturations and keep plant variables within safe limits. In recent years, several feedback control techniques of dynamic systems have been developed which are able to handle input and/or state-related constraints (Gilbert and Tan, 1991; Gilbert *et al.*, 1995; Bemporad *et al.*, 1997; Angeli *et al.*, 2001; Theilliol *et al.*, 2008a; Theilliol *et al.*, 2008b), using methods based on predictive control ideas, to synthesize the command or the Reference Governor (RG).

Kolmanovsky and Sun (2006) propose a Parameter Governor (PG) unit which enforces pointwise-in-time constraints on the evolutions of relevant system variables. Later, both RG and PG actions are integrated in a single unit as the Reference-Offset Governor (ROG) (Casavola *et al.*, 2007), which adds many advantages especially in enlarging the set of feasible evolutions of the system. The function of the ROG device is to modify, whenever necessary, the reference and add an offset to the nominal control action in order to enforce pointwise-in-time constraints and to improve the overall system transient performance (cf. Casavola *et al.*, 2006; 2007). The adaptation of the ROG with fault tolerant control systems is presented by Boussaid *et al.* (2010).

The paper is organized as follows. The reconfigurable control allocation and the reference governor issue

for actuator faults are presented in Section 2. The reformulation of the reconfigurable control allocation problem integrating the reliability requirements and the reference adaptation is proposed in Section 3. In Section 4, the proposed approach is illustrated and applied to a linearized aircraft model from the ADMIR simulator. Finally, concluding remarks are given in the last part of this work.

2. Reconfigurable control allocation and reference governor design

The proposed reconfigurable control system is composed of three modules: the reference governor module, the nominal controller and the allocation module. The benefit of such a structure (e.g., Fig. 1) is the possibility to deal with the largest range of faults that may occur in the system. For minor faults, the reconfiguration of the allocation module can compensate the effects of faults. Nevertheless, major faults should be treated carefully such as the saturation of the actuators. Thus, the reference governor is one solution to deal with actuator saturation by modifying the references to acceptable performances.

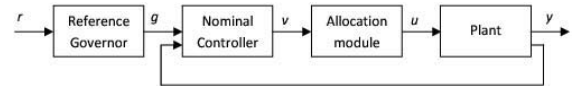


Fig. 1. Simplified structure of the proposed nominal system.

2.1. LTI formulation of control allocation. Let us consider the LTI system given by

$$\begin{cases} x(t+1) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$ are respectively the state, the control and the output matrices. $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the system output, and (A, B) is stabilizable. Control allocation is generally used for over-actuated systems, where the number of operable controls is greater than the controlled variables.

As defined by Harkegard and Glad (2005), while $\text{rank}(B) = q < m$ is satisfied, a virtual control input $v \in \mathbb{R}^q$ can be introduced and an equivalent description of (1) can be obtained as follows:

$$\begin{cases} x(t+1) = Ax(t) + B_v v(t), \\ v(t) = B^* u(t), \\ y(t) = Cx(t) \end{cases} \quad (2)$$

by considering the factorization

$$B = B_v B^*, \quad (3)$$

where $B_v \in \mathbb{R}^{n \times q}$ and $B^* \in \mathbb{R}^{q \times m}$.

In fact, $v(t)$ represents the set of the desired control efforts to be applied to the system in order to satisfy the performance requirements. It can be defined by solving based on the following linear quadratic optimization problem:

$$v(t) \rightarrow \min_u \int_0^\infty (x(t)^T Q_2 x(t) + v(t)^T R_2 v(t)) dt, \quad (4)$$

where $Q_2 \in \mathbb{R}^{n \times n}$ and $R_2 \in \mathbb{R}^{q \times q}$.

For simplicity and for this study, the case $q = p$, i.e., when the number of virtual controls equals the number of variables to be controlled, is considered.

Property 1. If the matrix B is not full rank, $\text{rank}(B) = q < n$, it can be factorized as $B = B_v B^*$, where $B_v \in \mathbb{R}^{n \times q}$, $B^* \in \mathbb{R}^{q \times m}$ and $\text{rank}(B^*) = q$. The virtual control input $v(t)$ can be determined in this case as $v(t) = B^* u(t)$, where $B^* = (B_v^T B_v)^{-1} B_v^T B$.

The control allocation problem can be expressed as a constrained linear mapping one based on the relationship,

$$v(t) = B^* u(t), \quad (5)$$

$$u_{\min} \leq u \leq u_{\max}, \quad (6)$$

where (6) is physical actuator saturation.

Optimization based control allocation methods aim at finding an optimal solution. If there is no exact solution, the optimal control is the feasible one such that $B^* u(t)$ approximates $v(t)$ as well as possible. The optimal control input can be obtained by a two-step optimization, namely, sequential quadratic programming:

$$\mathcal{U} = \arg \min_{u_{\min} \leq u \leq u_{\max}} \|B^* u - v\|_2, \quad (7)$$

$$u = \arg \min_{u \in \mathcal{U}} \|W_u u\|_2, \quad (8)$$

where \mathcal{U} is the set of feasible solutions subject to the cost criterion (8). The weighting matrix $W_u \in \mathbb{R}^{m \times m} \succ 0$ is used to give a specific priority level to the actuators.

In order to improve the safety of the system and preserve the actuators, a specific choice of the weighting matrix W_u is proposed based on actuator reliability indicators. The weighing matrix W_u is considered a key to manage the redundant actuators and contribute to a reliable controller improving system reliability. This technique can increase the life time of the system and prevent additional faults from occurring.

2.2. Reliability integration in control allocation design. Reliability $R(t)$ is defined as the probability that units, components, equipments and systems will accomplish the intended function for a specified period of

time under some stated conditions and specific environments (Gertsbakh, 2000).

In many situations, and especially in the present study, failure rates are obtained from components under different load levels. Several mathematical models have been developed to define the failure level in order to estimate the failure rate λ (Martorell *et al.*, 2009). The proportional hazard model introduced by Cox (1972) is used in this paper.

Definition 1. The failure rate is modeled as follows:

$$\lambda_i = \lambda_i^{bl} \mu_i(\ell, \vartheta), \quad (9)$$

where λ_i^{bl} represents the baseline failure rate (nominal failure rate) for the i -th subsystem or component and $\mu_i(\ell, \vartheta)$ is a function (independent of time) taking into account the effects of loads applied with ℓ presenting an image of the load and ϑ defining some parameters of the subsystem or component.

Definition 2. Different definitions of the load function $\mu_i(\ell, \vartheta)$ exist in the literature (Martorell *et al.*, 2009). However, the exponential form is commonly used (Guenab *et al.*, 2006). Moreover, the failure rate functions for the exponential distribution change according to the load level, assumed to be directly associated with the control input,

$$\mu_i(\ell, \vartheta) = \mu(\|u_i\|) = \exp(-\|u_i\|_2^2). \quad (10)$$

As given in (9), the failure rate of the actuator can be defined according to the load level, which is proportional to the control input applied,

$$\lambda_i \geq \lambda_i^{bl}, \quad i = 1, \dots, m. \quad (11)$$

Definition 3. For the exponential distribution of reliability, the mean time before the first failure (MTTF) can be adopted as a reliability indicator defined

$$\text{MTTF} = \int_0^\infty R(t) dt = \frac{1}{\lambda} \quad (12)$$

Moreover, for m redundant components, the overall system reliability can be computed at the end of the mission defined by $t = t_M$ as follows:

$$R_g(t_M) = 1 - \prod_{i=1}^m (1 - R_i(t_M)). \quad (13)$$

To implement the solution of the control allocation problem, and keep the set of the actuators available as long as possible, the desired efforts $v(t)$ defined by the controller can be distributed proportionally to the actuator reliability indicator. In order to consider such an approach, the values of the weighing matrix W_u are defined

as

$$W_u = \begin{pmatrix} \frac{\lambda_1^{bl}}{\lambda_{\max}^{bl}} & & & 0 \\ & \frac{\lambda_2^{bl}}{\lambda_{\max}^{bl}} & & \\ & & \ddots & \\ 0 & & & \frac{\lambda_m^{bl}}{\lambda_{\max}^{bl}} \end{pmatrix} \succ 0, \quad (14)$$

where $\lambda_{\max}^{bl} = \max(\lambda_i^{bl})$ is the upper failure rate corresponding to the least reliable actuator.

Consequently, for $\lambda_i^{bl} \ll \lambda_{\max}^{bl}$, $w_i \rightarrow 0$ and so the associated control component u_i^* (a solution of the optimization problem (8)) becomes very large. In addition, as $\lambda_i^{bl} \rightarrow \lambda_{\max}^{bl}$, $w_i \rightarrow 1$ and the associated control input is heavily weighted. The actuators are utilized in the control allocation proportionally to their health. This off-line synthesis of the control allocation strategy reduces the solicitation of the sensible actuators, which improves system safety and minimizes actuator aging. Indeed, the following relation can be achieved:

$$\{ \lambda_i^{bl} \rightarrow \lambda_{\max}^{bl} : u_i^* \rightarrow 0 \}, \quad (15)$$

where λ_{\max}^{bl} is the failure rate of the less reliable actuator.

2.3. LQ controller design. Let us consider the global system including the RG unit and the feedback controller, as depicted in Fig. 2. According to (2), and by

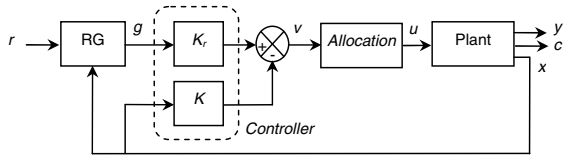


Fig. 2. Global system diagram including controller and reference governor blocks.

solving the Linear Quadratic Regulation (LQR) problem (Staroswiecki, 2003; Harkegard and Glad, 2005), the optimal control law is given by

$$v(t) = -K^*x(t) + K_r^*g(t), \quad (16)$$

where $g(t) \in \mathbb{R}^p$ is the manipulable reference, K^* and K_r^* the controller feedback and feed-forward gains, respectively, with

$$K^* = R^{-1}B_v^T S, \quad (17)$$

$$K_r^* = R^{-\frac{1}{2}}(C(B_v K^* - A)^{-1}B_v R^{-\frac{1}{2}})^+, \quad (18)$$

where $Q \in \mathbb{R}^{n \times n}$ is a positive semi-definite matrix and $R \in \mathbb{R}^{q \times q}$ is a positive definite matrix. Q and R are

preselected by the designer to achieve the nominal performance. S is a unique positive semi-definite and symmetric solution of the Algebraic Riccati Equation (ARE)

$$A^T S + SA + Q - SB_v R^{-1} B_v^T S = 0. \quad (19)$$

2.4. LTI formulation of the reference governor. As mentioned in Section 2.1 and according to (5), the control input $u(t)$ can be written as

$$u(t) = Pv(t) \quad (20)$$

with

$$P = W_u^{-1}(B^*W_u^{-1})^+.$$

Replacing (16) in (20), we can write

$$u(t) = -Kx(t) + K_g g(t), \quad (21)$$

where $K = PK^*$, $K_g = PK_r^*$, and $g(t) \in \mathbb{R}^p$ is the RG output, with $r(t) \in \mathbb{R}^p$ being the nominal reference.

Substituting (21) in (1), we obtain

$$x(t+1) = \Phi x(t) + Gg(t), \quad (22)$$

where $\Phi = (A - BK)$ and $G = BK_g$. We deal only with actuator saturations in this work, which means that we consider only constraints on the control inputs. Set $H_c = -K$ and $L = K_g$, so that the constraint vector $c(t)$ can be written as:

$$c(t) = H_c x(t) + Lg(t). \quad (23)$$

Thus, the LTI system in (1) becomes

$$\begin{cases} x(t+1) = \Phi x(t) + Gg(t), \\ y(t) = H_y x(t), \\ c(t) = H_c x(t) + Lg(t), \end{cases} \quad (24)$$

$c(t) \in \mathbb{R}^{n_c}$ being the constraint vector, $c(t) \in \mathcal{C}$; for any $t \in \mathbb{Z}_+$, and $\mathcal{C} \subset \mathbb{R}^{n_c}$ is a prescribed constraint set.

We make the following assumptions:

- A.1. Φ is a Stable matrix.
- A.2. The system (24) is offset-free with respect to $g(t)$, i.e., $H_y(I_n - \Phi)^{-1}G_g = I_p$.

The RG design problem consists in generating, at each time t , the command input $g(t)$ as an algebraic function of the current state $x(t)$ and reference $r(t)$,

$$g(t) := \bar{g}(x(t), r(t)). \quad (25)$$

The RG output is based on the minimization of a cost function subject to prescribed constraints. The cost func-

tion has the following form:

$$J(x(t), g(t), r) = \|g(t) - r\|_{\Psi_g}^2, \quad (26)$$

where $\Psi_g = \Psi_g^T > 0_m$ and $\|v\|_{\Psi}^2 := v^T \Psi v$. Thus, at each time $t \in \mathbb{Z}_+$, the RG output is chosen according to the solution of the following constrained optimization problem (cf. Casavola *et al.*, 2007):

$$g(t) := \arg \min_{g \in \mathcal{V}(x(t))} J(x(t), g(t), r), \quad (27)$$

where $\mathcal{V}(x(t))$ represents the set of all virtual sequences whose evolutions starting from $x(t)$ satisfy the constraints during transients and which is constructed from the recursive set developed by Casavola *et al.* (2007).

3. On-line control re-allocation and reference governor synthesis

In a degraded behavior and after a fault occurrence, the desired efforts are distributed among the actuators based on the re-estimation of their reliability indicators. The control inputs u_i , $i = 1, \dots, m$, are re-allocated taken into account actuator aging (see Fig. 3).

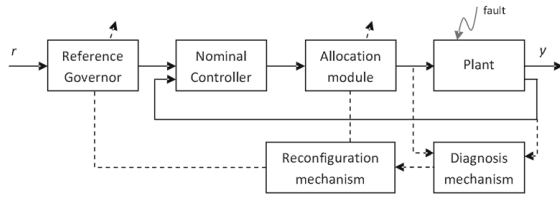


Fig. 3. Proposed structure of the reconfigurable control system.

Due to abnormal operation or material aging, actuator faults may occur and increase the complexity of the on-line control allocation problem. In this work, a loss of effectiveness control is considered where the system (2) can be written as a degraded functional as follows:

$$\begin{cases} x(t+1) = Ax(t) + B_v v(t), \\ v(t) = B_f^* u(t), \\ y(t) = Cx(t). \end{cases} \quad (28)$$

The matrix B_f^* can be written according to the nominal control input matrix B^* and the control effectiveness factors $\gamma_i \in [0, 1]$, $i = 1, \dots, m$, as follows:

$$B_f^* = B^*(I_m - \Gamma), \quad \Gamma = \begin{pmatrix} \gamma_1 & & 0 \\ & \gamma_2 & \\ & & \ddots \\ 0 & & & \gamma_m \end{pmatrix}.$$

Indeed, if $\gamma_i = 0$, then the i -th actuator is considered

in the fault-free case. Nevertheless, when $0 < \gamma_i < 1$, a fault which presents a partial loss of effectiveness control is considered. Moreover, when $\gamma_i = 1$, a failure is considered and the i -th actuator is out of order.

3.1. On-line reliability indicator estimation. The control redistribution in the faulty case requires the update of the weighting matrix and the estimation of the actuator reliability indicators according to the time of fault occurrence. In the following, a fault is assumed to be detected and isolated at time $t = t_f$. As presented previously, the MTTF is defined as the expected value of the failure distribution $V(t)$:

$$\text{MTTF} = 1 + \sum_{t=0}^{\infty} \prod_{j=1}^t (1 - \lambda(j)). \quad (29)$$

Indeed, for a reconfigurable system at $\tau = t_f + \Delta t$, the mean time before the first failure can be written according to $t = \tau$ as follows:

$$\text{MTTF}_{\tau} = 1 + \sum_{t=\tau}^{\infty} \prod_{j=1}^t (1 - \lambda(j)), \quad (30)$$

where MTTF_{τ} can be seen as an estimate of the mean time before a failure for a new reconfiguration. The failure rate corresponding to the rest of the component life time λ_i^f can be written as

$$\lambda_i^f = \frac{\lambda_i^0}{\lambda_i^0(\tau - 1)Te + 1} \exp(\lambda^0(\tau - 1)Te), \quad (31)$$

where λ^f is calculated according to the load level defined for $t \in [0, t_f]$ as in (9) and T_e is the sampling time.

In order to integrate actuator degradation in the reconfigured control allocation strategy, the control input can be obtained by solving the optimization problem (7) and (8) where, for $t \in [\tau, t_M]$, B^* is replaced by B_f^* and the weighting matrix W_u is re-estimated and changed on-line according to the new failure rates values λ_i^f ,

$$W_u = \begin{pmatrix} \frac{\lambda_1^f}{\lambda_{\max}^f} & & 0 \\ & \frac{\lambda_2^f}{\lambda_{\max}^f} & \\ & & \ddots \\ 0 & & & \frac{\lambda_m^f}{\lambda_{\max}^f} \end{pmatrix} \succ 0.$$

In fact, $\lambda_{\max}^f = \max(\lambda_i^f)$, $i = 1, \dots, m$, is the upper value of λ_i^f corresponding to the most degraded actuator.

Indeed, if an actuator fault occurs, the weighing matrix will be changed on-line W_u and a control input $u(t)$

is re-allocated smartly in order to minimize the use of the sensible actuators.

3.2. Pseudo-inverse method. In the faulty case, the control re-allocation problem consist in finding the control input $u(t)$ minimizing (8) and satisfying $B_f^*u(t) = v(t)$. If the above control constraint (6) is not considered and $u_d = 0$, an explicit solution can be obtained from minimization of the above quadratic problem (7) and (8) as follows:

$$\begin{aligned} \min_u J_w &= \|W_u u\|_2, \\ \text{s.t. } B_f^* u(t) &= v(t), \end{aligned} \quad (32)$$

while the solution is given based on a weighted pseudo-inverse as

$$u(t) = W_u^{-1}(B_f^*W_u^{-1})^+v(t). \quad (33)$$

where ‘+’ is the pseudo-inverse operator. Obviously, there is no guarantee that the solution will satisfy the constraints.

Improved approaches have been proposed to accommodate to the limits. The Redistributed Pseudo-Inverse (RPI) method proposed by Virnig and Bodden (1994) is an alternative solution, in which all control inputs that violate their bounds in the pseudo-inverse solution are saturated and removed from the optimization. Then the control problem is solved with only the remaining control inputs as free variables. The Cascaded Generalized Inverse (CGI) method proposed by (Bordignon, 1996) is an iterative redistributed pseudo-inverse. All control inputs that violate their bounds are considered to be saturated values and are removed at each step. The redistribution process is continued until either the pseudo-inverse solution is feasible or all control inputs are saturated. Enns (1998) also suggests to compute the pseudo-inverse solution iteratively, as in the CGI, but only to saturate one control input per iteration. For the proposed approach and in order to assure the reliability of the actuators, the most reliable actuator will be saturated first at each step and then removed from the CGI optimization problem.

3.3. Reference governor post-fault formulation. The state description of the plant in a closed loop scheme, after the fault diagnosis, is given by

$$\begin{cases} x(t+1) = \Phi_f x(t) + G_f g(t), \\ y(t) = H_y x(t), \\ c(t) = H_c^f x(t) + L_f g(t), \end{cases} \quad (34)$$

where Φ_f and G_f represent the global system dynamics after the fault occurrence:

$$\Phi_f = \Phi + B\Gamma K, \quad (35)$$

$$G_f = G - B\Gamma K_g. \quad (36)$$

Consider the RG unit in the faulty case, with the following assumptions:

- B1. Φ_f is a stable matrix.
- B2. The system (34) is offset-free with respect to $g(t)$, i.e., $H_y(I_n - \Phi_f)^{-1}G_g^f = I_p$.

The solution minimizing the cost function (26) is

$$g(t) := \arg \min_{g \in \mathcal{V}_f(x(t))} J(x(t), g(t), r), \quad (37)$$

with $\mathcal{V}_f(x(t))$ being the set of the disturbance-free virtual evolution of the constraint vector $\bar{c}_f(k, x(t), g)$ after the fault occurrence,

$$\begin{aligned} \mathcal{V}_f(x(t)) &= \left\{ g \in \mathcal{W}_\delta^f : \bar{c}_f(k, x(t), g) \in \mathcal{C}_k^f, \forall k \in \mathbb{Z}_+ \right\}, \end{aligned} \quad (38)$$

where $\bar{c}_f(k, x(t), g)$ is given by

$$\begin{aligned} \bar{c}_f(k, x(t), g) &= H_c^f (\Phi_f^k x(t) + \sum_{i=0}^{k-1} \Phi_f^{k-i-1} G_f g) + L_f g \end{aligned} \quad (39)$$

and

$$\mathcal{W}_\delta^f := \left\{ g \in \mathbb{R}^p : \bar{c}_g \in \mathcal{C}_f^\delta \right\}, \quad (40)$$

$$\mathcal{C}_f^\delta := \mathcal{C}_\infty^f \sim \mathcal{B}_\delta. \quad (41)$$

Here \mathcal{C}_∞^f is the prescribed constrained set after the fault occurrence, and \mathcal{B}_δ is a ball with radius δ centered at the origin. The following properties hold true for the above described RG in the faulty case.

Theorem 1. *Let Assumption B1 be satisfied. Consider the system (34) along with the RG selection rule (37), and let $\mathcal{V}_f(x(0))$ be non-empty. Then we have the following:*

1. *The minimizer in (37) uniquely exists at each $t \in \mathbb{Z}_+$ and can be obtained by solving a convex constrained optimization problem, viz. the fact that $\mathcal{V}_f(x(0)) = \mathcal{V}(x(t_f))$ is non-empty implies that $\mathcal{V}_f(x(t))$ is non-empty along the trajectories generated by the RG command (34). Such a time of fault occurrence t_f is determined by the diagnosis stage.*
2. *The set $\mathcal{V}_f(x(t))$ is determined for any, $x(t) \in \mathbb{R}^n$ where there exists an integer k_0^f such that, if $\bar{c}_f(k, x(t), g) \in \mathcal{C}_k^f, k \in \{0, 1, \dots, k_0^f\}$, then $\bar{c}_f(k, x(t), g) \in \mathcal{C}_k^f, \forall k \in \mathbb{Z}_+$. Such a constraint horizon k_0^f can be determined off-line as described by Gilbert and Tan (1991).*
3. *The constraints are fulfilled for all $t \in \mathbb{Z}_+$.*

4. The overall system is asymptotically stable. In particular, whenever $r(t) \equiv r$, and $g(t)$ converges either to r or to its best steady-state admissible approximation \hat{r} , with

$$\hat{g}(t) := \hat{r} := \arg \min_{g \in \mathcal{V}_f(x(t))} J(x(t), g(t), r). \quad (42)$$

Consequently, by the offset-free condition B2, we get $\lim_{t \rightarrow \infty} \bar{y}(t) = \hat{r}$, where \bar{y} is the disturbance-free component of y .

4. Flight control example

The ADMIRE model has been used by several researchers (e.g., Harkegard, 2003) and within the Group of Aeronautical Research and Technology in Europe (GARTEUR). The linear model used here has been obtained at a low speed flight condition of Mach 0.22 at an altitude of 3000 m and is similar to the one by Harkegard and Glad (2005). The states are $x = [\alpha \beta p q r]^T$ with controlled outputs $y = [\alpha \beta p]$, where α is the angle of attack (rad), β is the sideslip angle (rad), and p is the roll rate (rad/s), q defines the pitch rate (rad/s) and r is the yaw rate (rad/s). The control surfaces are $\sigma = [\sigma_c \sigma_{re} \sigma_{le} \sigma_r]^T$ and represent the deflections of the canard, right eleven, left eleven and rudder, respectively. Besides, the actuator position constraints are

$$\begin{aligned} \sigma_{\min} &= [-55^\circ \quad -30^\circ \quad -30^\circ \quad -30^\circ]^T, \\ \sigma_{\max} &= [25^\circ \quad 30^\circ \quad 30^\circ \quad 30^\circ]^T. \end{aligned}$$

A linearized model (Harkegard and Glad, 2005), for a sampling time of 0.5 s, is

$$\begin{aligned} A &= \begin{bmatrix} 1.0214 & 0.0054 & 0.0003 & 0.4176 & -0.0013 \\ 0 & 0.6307 & 0.0821 & 0 & -0.3792 \\ 0 & -3.4485 & 0.3979 & 0 & 1.1569 \\ 1.1199 & 0.0024 & 0.0001 & 1.0374 & -0.0003 \\ 0 & 0.3802 & -0.0156 & 0 & 0.8062 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.1823 & -0.1798 & -0.1795 & 0.0008 \\ 0 & -0.0639 & 0.0639 & 0.1397 \\ 0 & -1.5840 & 1.5840 & 0.2936 \\ 0.8075 & -0.6456 & -0.6456 & 0.0013 \\ 0 & -0.1005 & 0.1005 & -0.4114 \end{bmatrix}. \end{aligned}$$

In this example, the actuator dynamics are neglected, and the approximate model can be given where

$$B = B_v B^*$$

and

$$B_v = \begin{bmatrix} 0_{2 \times 3} \\ I_{3 \times 3} \end{bmatrix},$$

Table 1. Failure rates of elementary components.

Failure rates	
λ_1^0	$4e-3 \text{ min}^{-1}$
λ_2^0	$9e-4 \text{ min}^{-1}$
λ_3^0	$3e-3 \text{ min}^{-1}$
λ_4^0	$9e-4 \text{ min}^{-1}$

$$B^* = \begin{bmatrix} 0 & -1.5840 & 1.5840 & 0.2936 \\ 0.8075 & -0.6456 & -0.6456 & 0.0013 \\ 0 & -0.1005 & 0.1005 & -0.4114 \end{bmatrix}.$$

The resulting virtual control input $v(t)$ contains the angular accelerations in roll, pitch, and yaw produced by the control surfaces.

In order to illustrate the proposed approach in the short time window, we adapt the values of the actuator failure rates with the time of the discussed scenario. The failure rates are considered with a very huge value and given in Table 1.

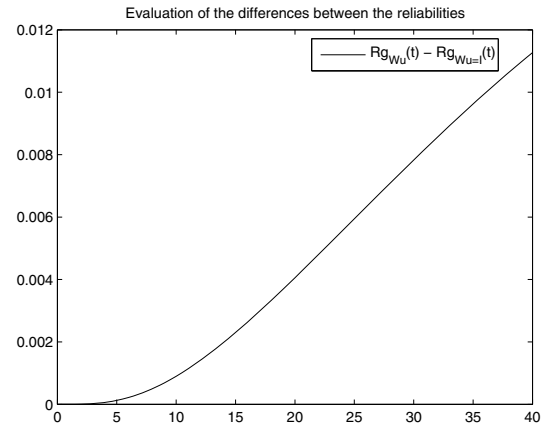


Fig. 4. Aircraft trajectory with re-allocation and the reference governor.

In this example, an optimal solution of the reconfigurable control problem is calculated in order to manage smartly the set of the actuators and increase the overall system reliability. The desired efforts are distributed against an optimal choice of the weighting matrix W_u based on the reliability indicators (14). With the proposed choice, the most reliable actuators defined by small failure rate values are more solicited in the effort distribution.

Figure 5 shows simulation results when a partial loss of effectiveness control is considered. In the considered scenario, the partial losses of effectiveness control correspond to $\gamma_3 = 0.3$. As shown for this scenario, the control re-allocation strategy is sufficient to compensate the effect of the considered fault where the signal references are not modified and the system outputs track the references. Indeed, it can be seen clearly in Fig. 6 that the desired efforts

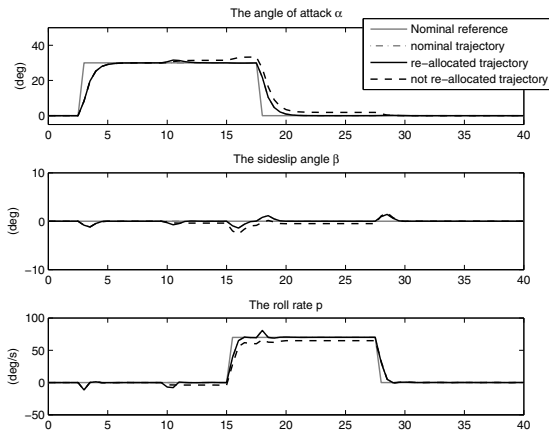


Fig. 5. Aircraft trajectory with re-allocation.

are distributed differently among the actuators taken into account the fault considered and the actuator reliability. This result can be justified by the fact that the analysed fault is relatively minor regarding the actuator availability.

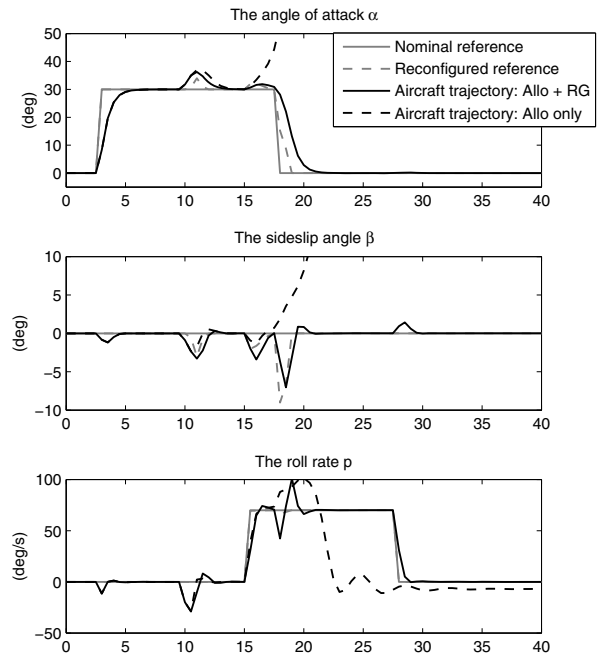


Fig. 7. Aircraft trajectory with re-allocation and the reference governor.

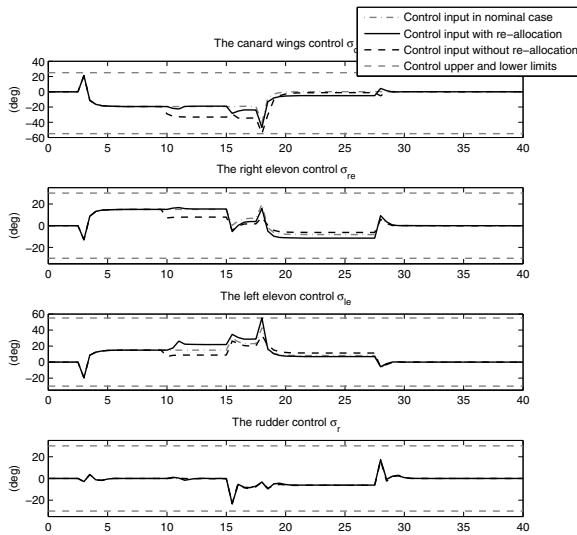


Fig. 6. Control input with re-allocation.

However, for a more severe degraded situation, the reference governor is needed to manage actuator saturation and tolerate the considered faults. Figure 7 shows the aircraft trajectories and the reconfigured references for $\gamma_3 = 0.85$. In fact, for this scenario the reference signals are modified in order to guarantee the system stability and distribute the desired efforts among the actuators with respect to system constraints. In Fig. 8, the efforts are distributed differently into account actuator reliability with respect to the examined fault and saturations.

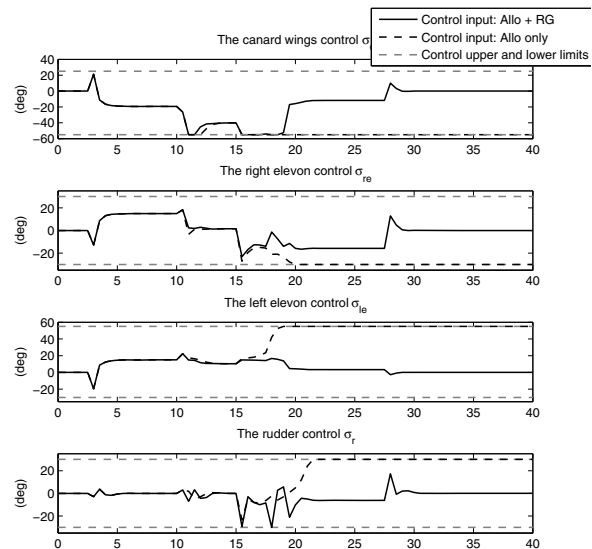


Fig. 8. Control input with re-allocation and the reference governor.

5. Conclusion

A contribution for an optimal reconfigurable control allocation strategy against reliability was proposed. The distribution of the desired efforts computed by the control law was considered based on actuator reliability. This paper presented a new approach to fault adaptation based on

reference governor combined with reconfigurable control allocation. For minor faults, control re-allocation can be a reliable solution to distribute the desired efforts with respect to actuator reliability by an optimal choice of the weighing matrix. However, for severely degraded situations, the RG can be used to solve the problem of stability and saturation also with respect to actuator reliability. The RG unit could modify the references and add an offset to the control inputs after fault occurrence and constraint violation in order to adapt the dynamics to the new special behavior which corresponds to faulty mode functioning. The simulation results show that these methods might improve the system performance and ensure safe plant functioning. Moreover, an optimal choice of the weighing matrix was proposed based on the characteristic of the implemented actuators. This strategy can preserve and improve the availability of actuators during a mission with high overall system reliability.

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