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A MIXED ACTIVE AND PASSIVE GLR TEST FOR A FAULT TOLERANT CONTROL SYSTEM

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This paper presents an adaptive Generalized Likelihood Ratio (GLR) test for multiple Faults Detection and Isolation (FDI) in stochastic linear dynamic systems. Based on the work of Willsky and Jones (1976), we propose a modified generalized likelihood ratio test, allowing detection, isolation and estimation of multiple sequential faults. Our contribution aims to maximise the good decision rate of fault detection using another updating strategy. This is based on a reference model updated on-line after each detection and isolation of one fault. To reduce the computational requirement, the passive GLR test will be derived from a state estimator designed on a fixed reference model directly sensitive to system changes. We will show that active and passive GLR tests will be mixed and give interesting results compared with the GLR of Willsky and Jones (1976), and that they can be easily integrated in a reconfigurable Fault-Tolerant Control System (FTCS) to asymptotically recover the nominal system performances of the jump-free system.

Keywords: generalized likelihood ratio, sequential jumps detection, two-stage Kalman filter, fault-tolerant control system.

1. Introduction

The diagnosis problem can be splitt into two steps: generation of residuals, which are ideally close to zero under no-fault conditions and minimally sensitive to noise, and residual evaluation, namely, the design of decision rules based on these residuals. This problem has been solved by many approaches: observers, parity space and fault detection filter. All these approaches are focused on residual generation, but are missing an appropriate test for a decision. In this work, we will develop a method which takes into account the residual generation problem based on a Kalman filter, associated with a GLR test decision for multiple faults.

The GLR test has been used in a wide variety of applications including the detection of sensor and actuator faults (Willsky, 1976; Willsky and Jones, 1976; Deckert *et al.*, 1977), electrocadiogram analysis (Gustafson *et al.*, 1978), geophysical signal processing (Basseville and Benveniste, 1983), and freeway supervision (Willsky *et al.*, 1980). For sequential fault detection in discrete-time stochastic linear systems, the GLR test includes the following steps:

1. Detection and isolation of one possible fault by applying a GLR detector for the innovation sequence of the Kalman filter designed on the jump-free system.
2. Updating the Kalman filter using the fault magnitude estimate by the GLR detector.
3. Go to Step 1 to detect, isolate and estimate another possible fault from measurements immediately available after the detection time of the last fault.

Following the notation used by Willsky (1986) or Basseville and Nikiforov (1994), the updating strategy of Willsky and Jones (1976), based on the incrementation of the state estimate $\hat{x}_k$ and its error covariance matrix P_k of the Kalman filter, proceeds as follows:

$$\hat{x}_k^{new} = \hat{x}_k^{old} + [\alpha_j(k,\hat{r}) - \beta_j(k,\hat{r})]\,\hat{\nu}(k,\hat{r}), \qquad (1)$$

$$P_k^{new} = P_k^{old} + [\alpha_j(k,\hat{r}) - \beta_j(k,\hat{r})]\,P^{\nu}(k,\hat{r}) \times [\alpha_j(k,\hat{r}) - \beta_j(k,\hat{r})]^T\,, \qquad (2)$$

where $(\hat{x}_k^{new}, P_k^{new})$ and $(\hat{x}_k^{old}, P_k^{old})$ represent respectively the new and old state estimates of the Kalman filter, $\alpha_j(k_j, \hat{r})$ and $\beta_j(k_j, \hat{r})$ are the fault signatures on the state and the state estimate, $\hat{r}$ is the estimated time of fault occurrence and $(\hat{\nu}(k, \hat{r}), P^{\nu}(k, \hat{r}))$ is the estimated fault magnitude produced by the GLR detector, immediately after the updating strategy (1). The innovation sequence of the resulting Kalman filter is given by

$$\gamma_k^{new} = \gamma_k^{old} - \rho_j(k, \hat{r})\hat{\nu}(k, \hat{r}), \tag{3}$$

$$H_k^{new} = H_k^{old} + \rho_j(k, \hat{r})P^{\nu}(k, \hat{r})\rho_j(k, \hat{r})^T. \tag{4}$$

Some criticism of this updating strategy includes what follows:

- What is the significant meaning of $(\hat{x}_k^{new}, P_k^{new})$ and $(\gamma_k^{new}, H_k^{new})$?
- Consequently, what are the stability and convergence conditions of the resulting Kalman filter?
- The threshold level of the GLR detector must be chosen to solve a trade-off between fast detection and accurate fault estimation.
- γ_k^{new} is not guaranteed to be a minimum variance white innovation sequence, which a necessary condition to minimize the rate of false alarms.

The first part of the paper presents an active GLR test. We will show that the updating strategies (1) and (2) have significant meaning for the Kalman filter designed on a new reference model including the original state vector of the system and the states of faults detected and isolated during the processing. The stability and convergence conditions of the augmented state Kalman filter designed on the new reference model will be established.

To reduce the computational requirement of the active GLR test, the second part of this paper presents a passive GLR test. Based on the augmented state Kalman filter designed on a fixed reference model including all the states of hypothetical faults at the beginning of the processing, the updating strategy will be based on the incrementation of the state estimate and the state estimate error covariance matrix after each detection of one fault as in the work of Willsky and Jones (1976). Less powerful than the active GLR test, we will show that it can be mixed with the active GLR test to derive a complete strategy allowing the treatment of the appearance and the disappearance of sequential faults. In the last part, we will show that the active and passive GLR tests can be easily integrated in a reconfigurable Fault-Tolerant Control System (FTCS) to asymptotically recover the nominal system performances of the fault-free system.

2. Active GLR test

The first part of this section classifies the meaning of the auxiliary innovation sequence (3) and (4) used in the modified GLR test by showing that (1) and (2) are included in the augmented state Kalman filter,

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{\nu}_{k+1} \end{bmatrix} = \hat{X}_{k+1} = \bar{A}\hat{X}_k + \bar{B}u_k + K_k\gamma_k, \tag{5}$$

$$\begin{aligned} \begin{bmatrix} P_{k+1}^x & P_{k+1}^{x\nu} \\ P_{k+1}^{\nu x} & P_{k+1}^{\nu} \end{bmatrix} &= \Omega_{k+1} \\ &= \bar{A}\Omega_k\bar{A}^T + \bar{\Gamma}W\bar{\Gamma}^T \\ &\quad - \bar{A}\Omega_k\bar{C}^T\left(\bar{C}\Omega_k\bar{C}^T + V\right)^{-1}\bar{C}\Omega_k\bar{A}^T, \end{aligned} \tag{6}$$

$$K_k = \begin{bmatrix} K_k^x \\ K_k^{\nu^j} \end{bmatrix} = \bar{A}\Omega_k\bar{C}^T H_k^{-1}, \quad H_k = \bar{C}\Omega\bar{C}^T + V, \tag{7}$$

designed on the new reference model h_j rewritten as

$$X_{k+1} = \bar{A}X_k + \bar{B}u_k + \bar{\Gamma}w_k, \tag{8}$$

$$y_k = \bar{C}X_k + v_k \tag{9}$$

with

$$X_k = \begin{bmatrix} x_k \\ \nu_k \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & f_j \\ 0 & 1 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \quad \bar{\Gamma} = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

where $\hat{x}_{k+1} = \hat{x}_{k+1}^{new}$ and $P_{k+1}^x = P_{k+1}^{new}$ represents the maximum likelihood prediction of the original state x_k but belonging now to the augmented state X_k.

Theorem 1. *In the two-stage Kalman filter of Friedland (1969), which optimally implements (5), the updating strategy rewritten as*

$$\hat{x}_{k+1} = \hat{\bar{x}}_{k+1} + \zeta_j(k+1, \hat{r})\hat{\nu}(k+1, \hat{r}), \tag{10}$$

$$P_{k+1} = \bar{P}_{k+1} + \zeta_j(k+1, \hat{r})P^{\nu}(k+1, \hat{r})\zeta_j(k+1, \hat{r})^T, \tag{11}$$

has the following meaning:

- $\left(\hat{\bar{x}}_{k+1}, \bar{P}_{k+1}\right)$ *is the state prediction of the jump-free system,*
- $(\hat{x}_{k+1}, P_{k+1})$ *is the reconfigured state prediction of the faulty system*
- $(\hat{\nu}(k+1, \hat{r}), P^{\nu}(k+1, \hat{r}))$ *is the prediction of the fault magnitude,*

and is optimal under extremely well estimated $\hat{r}$ if the augmented state Kalman filter (5) is correctly initialized at the detection time of the fault with

$$\hat{X}_k = \begin{bmatrix} I & \zeta_j(k, \hat{r}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\bar{x}}_k \\ \hat{\nu}(k, \hat{r}) \end{bmatrix}, \tag{12}$$

Ω_k

$$= \begin{bmatrix} I & \zeta_j(k,\hat{r}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{P}_k & 0 \\ 0 & P^\nu(k,\hat{r}) \end{bmatrix} \begin{bmatrix} I & \zeta_j(k,\hat{r}) \\ 0 & 1 \end{bmatrix}^T$$

from the quantities $\left(\hat{\bar{x}}_k, \bar{P}_k\right)$, $(\hat{\nu}(k,\hat{r}), P^\nu(k,\hat{r}))$ *and* $\zeta_j(k,\hat{r})$ *given by the GLR detector.*

Proof. At time $t_j = \hat{r} + \rho_j$, $(\hat{\nu})$ represents the minimum-time prediction of ν given by

$$\begin{aligned} &\hat{\nu}(t_j+1,\hat{r}) \\ &\quad = \left[\left(CA^{\rho_j-1}f_j\right)^T \bar{H}_{tj}^{-1}\left(CA^{\rho_j-1}f_j\right)\right]^{-1} \\ &\quad \times \left(CA^{\rho_j-1}f_j\right)^T \bar{H}_{tj}^{-1}\bar{\gamma}_{tj}, \end{aligned} \tag{13}$$

$$P^\nu(t_j+1,\hat{r}) = \left[\left(CA^{\rho_j-1}f_j\right)^T \bar{H}_{tj}^{-1}\left(CA^{\rho_j-1}f_j\right)\right]^{-1}, \tag{14}$$

under the assumption that ν has an infinite *a priori* covariance since $\varrho_j(k,\hat{r}) = 0$ for $k < t_j$ and $\varrho_j(t_j,\hat{r}) = CA^{\rho_j-1}f_j$. Thus, the updating strategy (10) and (11) applied at time t_j is given by

$$\hat{x}_{t_j+1} = \hat{\bar{x}}_{t_j+1} + \zeta_j(t_j+1,\hat{r}_j)\hat{\nu}(t_j+1,\hat{r}), \tag{15}$$

$$P_{t_j+1} = \bar{P}_{t_j+1} + \zeta_j(t_j+1,\hat{r})P^\nu(t_j+1,\hat{r})\zeta_j^T, \tag{16}$$

and can be used to define the Gaussian state prediction of the initial state X_{t_j+1} as

$$\hat{X}_{t_j+1} = \begin{bmatrix} \hat{x}_{t_j+1} \\ \hat{\nu}(t_j+1,\hat{r}) \end{bmatrix}, \tag{17}$$

Ω_{t_j+1}

$$= \begin{bmatrix} P_{t_j+1} & \zeta_j(t_j+1,\hat{r})P^\nu \\ P^\nu(t_j+1,\hat{r})\zeta_j(t_j+1,\hat{r})^T & P^\nu(t_j+1,\hat{r}) \end{bmatrix}.$$

The augmented state Kalman filter (5) can be implemented from the two-stage Kalman filter of Friedland (1969) (see Appendix) described by

$$\hat{x}_{k+1} = \hat{\bar{x}}_{k+1} + \zeta_{k+1}\hat{\nu}_{k+1}, \tag{18}$$

$$P_{k+1} = \bar{P}_{k+1} + \zeta_{k+1}P_{k+1}^\nu\zeta_{k+1}^T, \tag{19}$$

where $\left(\hat{\bar{x}}_{k+1}, \bar{P}_{k+1}\right)$ are given by the Kalman filter designed under h_0,

$$\hat{\bar{x}}_{k+1} = A\hat{\bar{x}}_k + Bu_k + \bar{K}_k(y_k - C\hat{\bar{x}}_k), \tag{20}$$

$$\bar{P}_{k+1} = A\bar{P}_kA^T + W - A\bar{P}_kC^T\bar{H}_k^{-1}C\bar{P}_kA^T, \tag{21}$$

$$\bar{K}_k = A\bar{P}_kC^T\bar{H}_k^{-1}, \tag{22}$$

$$\bar{H}_k = C\bar{P}_kC^T + V, \tag{23}$$

where $(\hat{\nu}_{k+1}, P_{k+1}^\nu)$ are given by the fault filter

$$\hat{\nu}_{k+1} = \hat{\nu}_k + K_k^\nu\gamma_k, \tag{24}$$

$$P_{k+1}^\nu = P_k^\nu - P_k^\nu\varrho_k^TH_k^{-1}\varrho_kP_k^\nu, \tag{25}$$

$$K_k^\nu = P_k^\nu\varrho_k^TH_k^{-1}, \tag{26}$$

$$\gamma_k = \bar{\gamma}_k - \varrho_k\hat{\nu}_k, \tag{27}$$

$$H_k = \bar{H}_k + \varrho_kP_k^\nu\varrho_k^T. \tag{28}$$

From the coupling equations

$$\zeta_{k+1} = (A - \bar{K}_kC)\zeta_k + f_j, \tag{29}$$

$$\varrho_k = C\zeta_k, \tag{30}$$

the initial values of the two-stage Kalman filter are

$$\begin{aligned} \begin{bmatrix} \hat{\bar{x}}_{t_j+1} \\ \hat{\nu}_{t_j+1} \end{bmatrix} &= \begin{bmatrix} I & -\zeta_{t_j+1} \\ 0 & 1 \end{bmatrix}\hat{X}_{t_j+1} \\ &= \begin{bmatrix} \hat{\bar{x}}_{t_j+1} \\ \hat{\nu}(t_j+1,\hat{r}) \end{bmatrix}, \end{aligned} \tag{31}$$

$$\begin{aligned} &\begin{bmatrix} \bar{P}_{t_j+1} & 0 \\ 0 & P_{t_j+1}^\nu \end{bmatrix} \\ &\quad = \begin{bmatrix} I & -\zeta_{t_j+1} \\ 0 & 1 \end{bmatrix}\Omega_{t_j+1}\begin{bmatrix} I & -\zeta_{t_j+1} \\ 0 & 1 \end{bmatrix}^T \\ &\quad = \begin{bmatrix} \bar{P}_{t_j+1} & 0 \\ 0 & P^\nu(t_j+1,\hat{r}) \end{bmatrix}, \end{aligned} \tag{32}$$

with $\zeta_{t_j+1} = \zeta_j(t_j+1,\hat{r})$.

After some manipulations, $\hat{\nu}$ can be rewritten in the form of a recursive filter,

$$\hat{\nu}(k+1,\hat{r}) = \hat{\nu}(k,\hat{r}) + K_k^\nu\gamma_k, \tag{33}$$

$$\begin{aligned} P^\nu(k+1,\hat{r}) &= P^\nu(k,\hat{r}) \\ &\quad - P^\nu\varrho_j^T(k,\hat{r})H_k^{-1}\varrho_jP^\nu, \end{aligned} \tag{34}$$

$$K_k^\nu = P^\nu(k,\hat{r})\varrho_j^T(k,\hat{r})H_k^{-1}, \tag{35}$$

$$\gamma_k = \bar{\gamma}_k - \varrho_j(k,\hat{r})\hat{\nu}_k, \tag{36}$$

$$H_k = \bar{H}_k + \varrho_j(k,\hat{r})P^\nu(k,\hat{r})\varrho_j^T(k,\hat{r}). \tag{37}$$

We can verify that (24) and (29) optimally implement (33), completing the proof of Theorem 1.

To avoid the trade-off between fast detection and accurate estimation, we conclude that the innovation sequence which must be used to detect, isolate and estimate a new fault is the innovation sequence of the fault filter (33), and (33) equals the auxiliary innovation sequence (1) and (2) used in the modified GLR test. This innovation sequence is also the innovation sequence of the augmented state Kalman filter guaranteed to be a minimum variance white innovation sequence allowing the design of a GLR detector. ■

Theorem 2. *After having initialized the augmented state Kalman filter (5) at the detection time of the first fault with the help of Theorem 1, another possible fault can be detected, isolated and estimated by the following GLR detector:*

$$\max_{\substack{i\in[1,\dots,N],\ i\neq j \\ \tilde{r}\in W}} \{T_i(k,\tilde{r}-\rho_i)\} > \varepsilon, \tag{38}$$

with

$$T_i^{new}(k,r) = b_i^{new}(k,r)^2 a_i^{new}(k,r)^{-1}, \quad (39)$$

$$a_i^{new}(k,r) = \sum_{t=r+\rho_i}^{k} [\varrho_i^{new}(t,r)]^T H_t^{-1} \varrho_i^{new}(t,r), \quad (40)$$

$$b_i^{new}(k,r) = \sum_{t=r+\rho_i}^{k} [\varrho_i^{new}(t,r)]^T H_t^{-1} \gamma_t, \quad (41)$$

where the new fault signatures $\varrho_i^{new}(t,r)$ are recursively computed as

$$\zeta_i^{new}(k+1,r) = (\bar{A} - \bar{K}_k \bar{C})\zeta_i^{new}(k,r) + \begin{bmatrix} f_i \\ 0 \end{bmatrix}, \quad (42)$$

$$\varrho_i^{new}(k,r) = \bar{C}\zeta_i^{new}(k,r), \quad (43)$$

$\zeta_i^{new}(r,r) = 0$*, where* $\zeta_i^{new}(t,r)$ *represents the additive effect of a new fault on the augmented state prediction error of the Kalman filter (57).*

Proof. The fault hypotheses denoted by h_i^{new} for $i \in [1,\dots,N]$ and $i \neq j$ can be modeled in relation with the new reference model (8) as

$$X_{k+1} = \bar{A}X_k + \bar{B}u_k + \begin{bmatrix} f_i(k,r) \\ 0 \end{bmatrix} \nu^{new}(k,r) + \bar{\Gamma}w_k, \quad (44)$$

$$y_k = \bar{C}X_k + v_k, \quad (45)$$

and can be confronted with the augmented state Kalman filter (5) as

$$h_j: \quad E(\gamma_t) = 0, \quad t < r \quad (46)$$

$$h_i^{new}: \quad E(\gamma_t) = \varrho_i^{new}(t,r)\nu^{new}, \quad k \geq t \geq r, \quad (47)$$

$i \in [1,\dots,N]$ and $i \neq j$, where the additive effect of h_i^{new} on its state prediction error

$$\begin{bmatrix} e_k^x \\ e_k^\nu \end{bmatrix} = X_k - \hat{X}_k$$

and on its innovation sequence $\gamma_k = y_k - \bar{C}\hat{X}_k$ is described by (42) and (43).

We have

$$E\{(\gamma_{k-t} - E(\gamma_{k-t}))(\gamma_k - E(\gamma_k))^T\} = 0, \quad \forall t < k \quad (48)$$

and $\varrho_i^{new}(r,r) = 0$, and so until $\varrho_i^{new}(r+\rho_i-1,r) = 0$, where $\varrho_i^{new}(r+\rho_i,r) = CA^{\rho_i-1}f_i$ (the detectability indexes ρ_i have not lost their significant meaning). Hence, the likelihood ratio between $h_i^{new}(i \neq j)$ and h_j is

$$\lambda_i^{new}(k,r,\nu^{new}) = \frac{\exp\left(-\frac{1}{2}\sum_{t=r+\rho_i}^{k} \|\gamma_t - \varrho_i^{new}(t,r)\nu^{new}\|^2_{\bar{H}_t^{-1}}\right)}{\exp\left(-\frac{1}{2}\sum_{t=r+\rho_i}^{k} \|\gamma_t\|^2_{\bar{H}_t^{-1}}\right)}. \quad (49)$$

Based on measurements up to time k, the maximum likelihood prediction of ν^{new} conditioned on r is

$$\hat{\nu}^{new}(k+1,r) = \left[\sum_{t=r+\rho_i}^{k} \varrho_i^{new}(t,r)^T \bar{H}_t^{-1} \varrho_i^{new}(t,r)\right]^{-1} \times \sum_{t=r+\rho_i}^{k} \varrho_i^T(t,r)\bar{H}_t^{-1}\gamma_t. \quad (50)$$

Substituting (50) in (49), we obtain the log-likelihood ratio

$$T_i^{new}(k,r) = 2\log(\lambda_i^{new}(k,r,\hat{\nu}^{new}(k+1,r))). \quad (51)$$

Thus, if

$$\max_{i\in[1,\dots,N],\ \tilde{r}\in[0,\dots,k]} \{T_i^{new}(k,\tilde{r}-\rho_i)\} > \varepsilon,$$

then a new fault is detected and isolated from $(j,\hat{\tilde{r}}) = \arg\max\{T_i^{new}(k,\tilde{r}-\rho_i)\}$ and its estimate is given by

$$\hat{\nu}(k+1,\hat{r}) = a_j^{new}(k,\hat{r})^{-1} b_j^{new}(k,\hat{r}), \quad (52)$$

$$P^\nu(k+1,\hat{r}) = a_j^{new}(k,\hat{r})^{-1} \quad (53)$$

with $\hat{r} = \hat{\tilde{r}} - \rho_j$, which completes the proof. ■

Theorem 3. *The first step of the active GLR test described by Theorems 1 and 2 follows the minimax strategy developed by Basseville and Nikiforov (1994). The Kullback divergence between h_i^{new} and h_j given by*

$$\delta_i^{new}(k,r) = \sum_{t=r}^{k} \left[\varrho_i^{new}(t,r)^T H_t^{-1} \varrho_i^{new}(t,r)\right] (\nu^{new})^2 \quad (54)$$

is maximized with respect to ν^{new} and satisfies $\delta_i^{new}(k,r) \geq \tilde{\delta}_i(k,r)$, where

$$\tilde{\delta}_i(k,r) = \sum_{t=r}^{k} \left[\varrho_i(t,r)^T H_t^{-1} \varrho_i(t,r)\right] (\nu^{new})^2 \quad (55)$$

is the Kullback divergence derived from the modified GLR test. The rate of good decisions will then be always superior to those obtained by the modified GLR test.

Proof. From the equations of the Kalman filter, the fault hypotheses h_i^{new} can be confronted as

$$h_j : \bar{\gamma}_t = \varrho_t \nu^{old}, \quad t < r, \tag{56}$$

$$h_i^{new} : \bar{\gamma}_t = \begin{bmatrix} \varrho_i(t,r) & \varrho_t \end{bmatrix} \begin{bmatrix} \nu^{new} \\ \nu^{old} \end{bmatrix}, \tag{57}$$

$k \geq t \geq r + \rho_i,\ \ i \in [1, \dots, N]$ for $i \neq j$, where ν^{old} can be viewed as a nuisance parameter. Using the optimal prediction of ν^{old} under h_j and h_i^{new} given by $\hat{\nu}_{t+1}$ and $\hat{\nu}_{t+1} + \zeta_i^\nu(t+1,r)\nu^{new}$, respectively, where $\zeta_i^\nu(t+1,r)$ describes the additive effect of the new fault on the bias filter (33) given by

$$\zeta_i^\nu(t+1,r) = (I - K_t^\nu \varrho_t)\zeta_i^\nu(t,r) - K_t^\nu \varrho_i(t,r) \tag{58}$$

with $\zeta_i^\nu(r,r) = 0$, the fault hypotheses (56) and (57) can be equivalently confronted as

$$\begin{aligned} h_j : &\quad \bar{\gamma}_t = \varrho_t \hat{\nu}_{t+1}, \qquad t < r, \\ h_i^{new} : &\quad \bar{\gamma}_t = \begin{bmatrix} \varrho_i(t,r) & \varrho_t \end{bmatrix} \begin{bmatrix} \nu^{new} \\ \hat{\nu}_{t+1} + \zeta_i^\nu \nu^{new} \end{bmatrix}, \end{aligned} \tag{59}$$

$t \geq r + \rho_i,\ \ i \in [1, \dots, N]$ for $i \neq j$, or

$$h_j : \quad E\left(\bar{\gamma}_t - \varrho_t \hat{\nu}_{t+1}\right) = 0, \qquad t < r, \tag{60}$$

$$h_i^{new} : \quad E\left(\bar{\gamma}_t - \varrho_t \hat{\nu}_{t+1}\right) = \left[\varrho_i + \varrho_t \zeta_i^\nu\right] \nu^{new}, \tag{61}$$

$t \geq r + \rho_i, i \in [1, \dots, N], i \neq j.$

The likelihood ratio between (60) and (61) gives

$$\begin{aligned} &\lambda_i^{new}(k, r, \nu^{new}) \\ &= \Big[\exp\big(- \frac{1}{2} \sum_{t=r+\rho_i}^{k} \|\big(I - \varrho_t K_t^\nu\big)\big(\bar{\gamma}_t - \varrho_t \hat{\nu}_t\big)\|^2_{Q_t^{-1}}\big) \\ &\quad \times \exp\big(- \frac{1}{2} \sum_{t=r+\rho_i}^{k} \|\big(I - \varrho_t K_t^\nu\big)\big(\bar{\gamma}_t - \varrho_t \hat{\nu} \\ &\quad - \left[\varrho_i + \varrho_t \zeta_i^\nu\right]\nu^{new}\|^2_{Q_t^{-1}}\big)\big)\Big]^{-1}, \end{aligned} \tag{62}$$

where

$$Q_t = (I - \varrho_t K_t^\nu) H_t (I - \varrho_t K_t^\nu)^T \tag{63}$$

since

$$\bar{\gamma}_t - \varrho_t \hat{\nu}_{t+1} = \left(I - \varrho_t K_t^\nu\right)\left(\bar{\gamma}_t - \varrho_t \hat{\nu}_t\right), \tag{64}$$

$$\begin{aligned} &\varrho_i(t,r) + \varrho_t \zeta_i^\nu(t+1,r) \\ &\qquad = \left(I - \varrho_t K_t^\nu\right)\left(\varrho_i + \varrho_t \zeta_i^\nu\right), \end{aligned} \tag{65}$$

or

$$\begin{aligned} &\lambda_i(k, r, \nu^{new}) \\ &\quad = \exp\Big(- \frac{1}{2} \sum_{t=r+\rho_i}^{k} \|\big(\bar{\gamma}_t - \varrho_t \hat{\nu}_t \\ &\quad - \left[\varrho_i + \varrho_t \zeta_i^\nu\right]\nu^{new}\big)\|^2_{H_t^{-1}}\Big) \\ &\quad \times [\exp\big(- \frac{1}{2} \sum_{t=r+\rho_i}^{k} \|\big(\bar{\gamma}_t - \varrho_t \hat{\nu}_t\big)\|^2_{H_t^{-1}}\big)]^{-1}. \end{aligned} \tag{66}$$

From the transformation

$$T_k = \begin{bmatrix} I & -\zeta_k \\ 0 & I \end{bmatrix}$$

used in Appendix, let

$$T_k \zeta_i^{new}(k,r) = \begin{bmatrix} \zeta_i(k,r) \\ \zeta_i^\nu(k,r) \end{bmatrix}.$$

Thus, the new fault signatures (42) can then be equivalently computed as

$$\begin{aligned} T_{k+1}\zeta_i^{new}(k+1,r) &= s(k,r) + T_{k+1}\begin{bmatrix} f_i \\ 0 \end{bmatrix}, \\ \varrho_i^{new}(k,r) &= \bar{C} T_k^{-1} T_k \zeta_i^{new}(k,r), \\ s(k,r) &= T_{k+1}(\bar{A} - K_k \bar{C}) T_k^{-1} \\ &\quad \times T_k \zeta_i^{new}(k,r), \end{aligned} \tag{67}$$

leading to

$$\begin{aligned} \begin{bmatrix} \zeta_i(k+1,r) \\ \zeta_i^\nu(k+1,r) \end{bmatrix} &= M \begin{bmatrix} \zeta_i(k,r) \\ \zeta_i^\nu(k,r) \end{bmatrix} + \begin{bmatrix} f_i \\ 0 \end{bmatrix}, \\ \varrho_i^{new}(k,r) &= C \begin{bmatrix} I & \zeta_k \end{bmatrix} \begin{bmatrix} \zeta_i(r,r) \\ \zeta_i^\nu(r,r) \end{bmatrix}, \end{aligned} \tag{68}$$

with

$$\begin{aligned} M = &\begin{bmatrix} A - \bar{K}_k C & 0 \\ -K_k^\nu C & I - K_k^\nu \varrho_k \end{bmatrix} \begin{bmatrix} \zeta_i(k,r) \\ \zeta_i^\nu(k,r) \end{bmatrix} \\ &+ \begin{bmatrix} f_i \\ 0 \end{bmatrix}, \end{aligned}$$

$$\begin{bmatrix} \zeta_i(r,r) \\ \zeta_i^\nu(r,r) \end{bmatrix} = 0,$$

where (68) gives $\varrho_i^{new}(k,r) = \varrho_i(k,r) + \varrho_k \zeta_i^\nu(k,r)$.

We conclude that (66) is equivalent to (49). From the two-stage Kalman filter results, we can verify that $P_i^\nu(k+1,r) = a_i^{new}(k,r)^{-1}$ satisfying the following Riccati difference equation:

$$\begin{aligned} &\begin{bmatrix} \Omega & 0 \\ 0 & P_i^\nu(k+1,r) \end{bmatrix} \\ &= \begin{bmatrix} \bar{A}\Omega\bar{A}^T + \bar{W} - K_k \bar{C}\Omega_k \bar{A}^T & 0 \\ 0 & \left[I - K_i^\nu \varrho_i^{new}\right] P_i^\nu \end{bmatrix}, \end{aligned} \tag{69}$$

where

$$K_i^\nu(k,r) = P_i^\nu \varrho_i^{newT}\big[\bar{C}\Omega_k\bar{C}^T + V + \varrho_i^{new}P_i^\nu\varrho_i^{newT}\big]^{-1}$$

minimizes the trace of $P_i^\nu(k+1,r)$ (and where the gain of the augmented state Kalman filter K_k minimized the trace of Ω_{k+1}), and the Kullback divergence $\delta_i^{new}(k,r) = [P_i^\nu(k,r)]^{-1}(\nu^{new})^2$ is then maximized with respect to ν^{new}. We have that

$$\begin{bmatrix} \Omega_{k+1} & 0 \\ 0 & P_i^\nu(k+1,r) \end{bmatrix}$$

is equivalent to

$$\begin{bmatrix} \bar{P}_{k+1} & 0 & 0 \\ 0 & P_{k+1}^\nu + \zeta_i^\nu P_i^\nu \zeta_i^{\nu T} & \zeta_i^\nu P_i^{\nu-1} \\ 0 & P_i^{\nu-1}\zeta_i^{\nu T} & P_i^\nu \end{bmatrix}. \quad (70)$$

From (70), the Kullback divergence between h_i^{new} and h_0 can be expressed as

$$\begin{bmatrix} \nu^{old} \\ \nu^{new} \end{bmatrix}^T \begin{bmatrix} P_{k+1}^\nu + \zeta_i^\nu P_i^\nu \zeta_i^{\nu T} & \zeta_i^\nu P_i^{\nu-1} \\ P_i^{\nu-1}\zeta_i^{\nu T} & P_i^\nu \end{bmatrix}^{-1} \begin{bmatrix} \nu^{old} \\ \nu^{new} \end{bmatrix}$$

$$= \begin{bmatrix} \nu^{old} - \zeta_i^\nu \nu^{new} \\ \nu^{new} \end{bmatrix}^T \begin{bmatrix} P_{k+1}^\nu & 0 \\ 0 & P_i^\nu \end{bmatrix}^{-1} \times \begin{bmatrix} \nu^{old} - \zeta_i^\nu \nu^{new} \\ \nu^{new} \end{bmatrix}. \quad (71)$$

The Kullback divergence (71) attains its minimal value $\delta_i^{new}(k,r) = [P_i^\nu(k+1,r)]^{-1}(\nu^{new})^2$ for $\nu^{old} = \zeta_i^\nu(k+1,r)\nu^{new}$. Hence we conclude that the first step of the active GLR test follows a minimax strategy (see Appendix), which completes the proof of Theorem 3. Based on an inductive reasoning with the help of Theorems 1 and 2, the proposed active GLR test is then derived leading to a GLR detector of the form

$$\max_{i\in[1,\dots,N], i\neq[\text{jumps already treated}]\ \tilde{r}\in W} \{T_i^{new}(k, \tilde{r}-\rho_i)\} > \varepsilon, \quad (72)$$

where the state vector

$$X_k = \begin{bmatrix} x_k^T & (\nu^{old})^T \end{bmatrix}^T$$

of the reference model (44) includes q states of faults

$$\nu_k^{old} = \begin{bmatrix} \nu_k^1 & \dots & \nu_k^q \end{bmatrix}^T$$

detected and isolated during the recursive processing.

In the work of Basseville and Nikiforov (1994), the off-line statistical decoupling of nuisance parameters is reduced to a static decoupling problem in a regression model. Our active GLR test solves on-line a dynamic statistical decoupling problem by rejecting the nuisance parameters which are statistically significant (see also Appendix). Under the multiple faults detectability and distinguishability conditions of Theorem 1, the augmented state Kalman filter is guaranteed to be stable at each step of the recursive treatment. With this implementation, the estimation of faults detected and isolated during the processing will be improved from measurements available after their detection. In the case where the old faults are extremely well estimated (the fault prediction errors do not converge exponentially to zero as the state prediction of the jump-free system but only asymptotically), then $\delta_i^{new}(k,r) = \tilde{\delta}_i(k,r)$ and the GLR test coincides with the modified GLR test. ■

3. Passive GLR test

The passive GLR test is based on the assumption that faults occur frequently. Hence, assume that the fixed reference model denoted by H_N is described as

$$X_{k+1} = \bar{A}X_k + \bar{B}u_k + \bar{\Gamma}w_k, \quad (73)$$

$$y_k = \bar{C}X_k + v_k, \quad (74)$$

with

$$\bar{A} = \begin{bmatrix} A & F \\ 0 & 1 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$\bar{\Gamma} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \qquad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$

$$X_k = \begin{bmatrix} x_k \\ \nu_k \end{bmatrix},$$

the augmented state model including all jump states

$$\nu_k = \begin{bmatrix} \nu_k^1 & \dots & \nu_k^j & \dots & \nu_k^N \end{bmatrix}$$

that we wish to detect and isolate. From (73) and (74), the described fault hypothesis h_j can be viewed as an impulsive abrupt change in the j-th hypothetical jump state, modeled as

$$\nu_{k+1}^j = \nu_k^j + \Delta\nu\delta_{kr}, \quad \forall j \in [1,\dots,N], \quad (75)$$

where r is the unknown occurrence time of the impulsive abrupt change, $\Delta\nu$ is the jump state increment and δ_{kr} is the Kronecker operator. Substituting (75) in (73), we obtain the impulsive fault hypotheses, denoted by h_j^Δ, described as

$$X_{k+1} = \bar{A}X_k + \bar{B}u_k + f_j^\Delta(k,r)\Delta\nu^j(k,r) + \bar{\Gamma}w_k,$$
$$y_k = \bar{C}X_k + v_k, \quad (76)$$

with $\Delta\nu^j(k,r) = \Delta\nu^j\delta_{kr_i}$, $f_j^\Delta(k,r) = f_j^\Delta\delta_{kr_i}$, where

$$f_j^\Delta = \begin{bmatrix} 0 \\ I_j \end{bmatrix}$$

and

$$I_j^T = \begin{bmatrix} 0 & \dots & 1 & \dots & 0 \end{bmatrix}$$

has unity in the j-th position and zero elsewhere.

Based on an approach very similar to the modified GLR test, is the augmented state Kalman filter designed on the reference model (73) and (74).

$$\begin{aligned}\hat{X}_{k+1} &= \bar{A}\hat{X}_k + \bar{B}u_k + K_k(y_k - \bar{C}\hat{X}_k),\\ \Omega_{k+1} &= \bar{A}\Omega_k\bar{A}^T + \bar{\Gamma}W\bar{\Gamma}^T\\ &\quad - \bar{A}_k\Omega_k\bar{C}^T\left(H_k\right)^{-1}\bar{C}\Omega_k\bar{A}^T, \qquad (77)\\ K_k &= \bar{A}\Omega_k\bar{C}^T H_k^{-1},\\ H_k &= \bar{C}\Omega_k\bar{C}^T + V,\end{aligned}$$

The additive effect of the impulsive jump hypotheses h_j^Δ on the state prediction error and on the innovation sequence of the augmented state Kalman filter propagates as

$$e_{k+1} = \tilde{e}_{k+1} + \zeta_j^\Delta(k+1,r)\Delta\nu, \qquad (78)$$

$$\gamma_k = \tilde{\gamma}_{k+1} + \varrho_j^\Delta(k,r)\Delta\nu, \qquad (79)$$

where $\tilde{e}_{k+1}$ and $\tilde{\gamma}_k$ represent the state prediction error and the innovation sequence on the jump-free system, and $\zeta_j^\Delta(k+1,r)$ and $\varrho_j^\Delta(k,r)$ propagate as

$$\begin{aligned}\zeta_j^\Delta(k+1,r) &= (\bar{A} - \bar{K}_k\bar{C})\zeta_j^\Delta(k,r),\\ \zeta_j^\Delta(r,r) &= f_j^\Delta,\\ \varrho_j^\Delta(k,r) &= \bar{C}\zeta_j^\Delta(k,r), \qquad (80)\end{aligned}$$

Thus, we can apply the following GLR detector:

$$\max_{j\in[1,\dots,N],\ \tilde{r}\in W}\{T_j^\Delta(k-\rho_j,\tilde{r})\} > \varepsilon, \qquad (81)$$

with

$$T_j^\Delta(k,r) = b_j^\Delta(k,r)^2 a_j^\Delta(k,r)^{-1}, \qquad (82)$$

$$a_j^\Delta(k,r) = \sum_{t=r+\rho_j}^{k}\varrho_j^{\Delta T}(t,r)H_t^{-1}\varrho_j^\Delta(t,r), \qquad (83)$$

$$b_j^\Delta(k,r) = \sum_{t=r+\rho_j}^{k}\varrho_j^{\Delta T}(t,r)H_t^{-1}\gamma_t. \qquad (84)$$

If

$$\max_{j,\ \tilde{r}}\{T_j^\Delta(k-\rho_j,\tilde{r}\} > \varepsilon,$$

then

$$(i,\hat{\tilde{r}}) = \arg\max_{j,\tilde{r}}\{T_j^\Delta(k,\tilde{r}-\rho_j)\}$$

and the impulsive fault h_i^Δ is declared to occur at the time when $\hat{r} = \hat{\tilde{r}} - \rho_i$, with

$$\Delta\hat{\nu}(k+1,\hat{r}) = a_i^\Delta(k,\hat{r})^{-1}b_i^\Delta(k,\hat{r}), \qquad (85)$$

$$P^{\Delta\nu}(k+1,\hat{r}) = a_i^\Delta(k,\hat{r})^{-1} \qquad (86)$$

representing the maximum likelihood prediction of the fault increment $\Delta\nu$ (under the assumption that $\Delta\nu$ has an infinite *a priori* covariance). At the detection time of the first fault, the tracking ability of the augmented state Kalman filter (129) can be improved from the updating strategy as

$$\begin{aligned}\hat{X}_{k+1}^{new} &= \hat{X}_{k+1}^{old} + \zeta_i^\Delta(k+1,\hat{r})\Delta\hat{\nu}(k+1,\hat{r}),\\ \Omega_{k+1}^{new} &= \Omega_{k+1}^{old}\\ &\quad + \zeta_i^\Delta(k+1,\hat{r})P^{\Delta\nu}(k+1,\hat{r})\zeta_i^\Delta(k+1,\hat{r})^T\end{aligned} \qquad (87)$$

In our case, the state of the matched filters given by $\zeta_j^\Delta(k,r)$ is spanned in the trajectory space of the prediction errors of the augmented state Kalman filter. Thus, (87) substituted in the augmented state Kalman filter (77) improves its tracking ability without producing a possible instability on the resulting filter (under the stability and convergence conditions of the augmented state Kalman filter given by Jamouli (2007)). The treatment of another impulsive fault is then obtained by applying the GLR detector (81) to the resulting filter after having reinitialized $\zeta_j^\Delta(k,r) = 0$, $\forall j \in [1,\dots,N]$ immediately after the filter incrementation. The new initialization (139) allows $E(\hat{X}_k^{new})$ to reach the true system state very quickly (and $E(\gamma_t)$ to reach zero for fault compensation, consequently) avoiding the detection of the same fault several times. From inductive reasoning, the passive GLR test is then derived and consists of the following steps:

1. Detection, isolation and estimation of one impulsive fault with the GLR detector (81).
2. Updating the augmented state Kalman filter (77) with (87) to improve its tracking.
3. Go to Step 1.

Sequential multiple decision theory is not complete and the choice of the threshold level ε is not studied in this paper. However, some simulation results not presented in this paper show that only the statistical tuning parameter ε can be fixed at the beginning of the processing (this is not the threshold level which is adaptive, but the augmented Kalman filter). If the updated reference model (8) is substituted into the fixed reference model (73), the fault hypothesis h_i^Δ can model another jump among the old changes or also the disappearance of the old jumps. In this case, a mixed active/passive GLR test can be derived for a complete strategy allowing the treatment of the appearance and the disappearance of sequential faults.

4. Reconfigurable fault-tolerant control system

The purpose of this section is to show how the active and passive GLR test can be used in an FTCS. A Fault-Tolerant Control System (FTCS) is a control system that possesses the ability to accommodate system component failures automatically. The existing methods for reconfigurable controller design include a linear quadratic regulator (Looze *et al.*, 1985), eigenstructure assignment (Jiang, 1994), a multiple model (Maybeck and Stevens, 1991), set-membership approaches (Puig, 2010), adaptive control (Bodson and Groszkiewicz, 1997), a pseudo-inverse (Caglayan, 1988) and model following (Huang and Stengel, 1990).

Recently, Mahmoud (2008; 2009) and Rafi *et al.* (2010) proposed a kind of stabilizing controllers for fault tolerant control. For fuzzy systems (Tong *et al.*, 2008), a new FTC approach is developed taking into account uncertainties in system models (Rodrigues, 2007). In general, an FTCS works as follows: a suitable Fault Detection and Isolation (FDI) strategy identifies the faults and their estimates are used to generate additional input signals which are superimposed on the nominal control inputs in such a way that the influence of the faults on the regulated variables is rejected.

To integrate the standard GLR test in an FTCS, Willsky (1976) proposed a control law of the form $u_k = -L\hat{x}_k^{new}$. To do the same with our active GLR test, Section 5 proposes the design of a Linear Quadratic Gaussian (LQG) regulator (Anderson and Moore, 1990) of the form $u_k = -L\hat{X}_k$, where $\hat{X}_k$ will be the state prediction of the updated reference model, thus reconfigured on-line after each detection and isolation of one fault.

Our FTCS is only designed to reach the unique goal

$$\lim_{k\to\infty} E(y_k) = 0 \quad \text{subject to} \quad r < \infty, \tag{88}$$

or, in other words, to asymptotically reject the effect of faults on the system output (the reference input will be maintained equal to zero, avoiding the need for a reconfigurable feedforward control law). The proposed FTCS is based on the active GLR test integrated via a reconfigurable control law designed on the model

$$X_{k+1} = \bar{A}X_k + \bar{B}u_k + \bar{\Gamma}w_k, \tag{89}$$

$$y_k = \bar{C}X_k + v_k, \tag{90}$$

where the main problem in reaching our goal is that the pair $(\bar{A}, \bar{B})$ has N uncontrollable modes (under controllable (A, B)). The reconfigurable control law of the form $u_k = u_k^n - G\hat{\nu}_k$ will be designed in such a way that the nominal control $u_k^n = -\bar{L}\hat{\bar{x}}_k$ of the jump-free system (obtained by an LQG approach on an infinite horizon) is reconfigured on-line after each detection and isolation of one impulsive fault by the additive term $G\hat{\nu}_k$.

In order to design G in relation with the available nominal control law, we assume that the implementation of the active GLR test is based on a two-stage Kalman filter, the only optimal filter which gives the state prediction of the jump-free system $\hat{\bar{x}}_k$. Thus, let

$$u_k = u_k^n - G\nu_k, \tag{91}$$

be the control law that we wish to design for a physical rejection of faults ν_k. Under the state transformation

$$\begin{bmatrix} \bar{x}_k \\ \nu_k \end{bmatrix} = \begin{bmatrix} I & T \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k \\ \nu_k \end{bmatrix}, \tag{92}$$

the system (89) with (92) can be expressed as

$$\begin{bmatrix} \bar{x}_{k+1} \\ \nu_{k+1} \end{bmatrix} = \begin{bmatrix} A & (I-A)T + F \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ \nu_k \end{bmatrix} \tag{93}$$

$$+ \begin{bmatrix} B \\ 0 \end{bmatrix} (u_k^n - G\nu_k), \tag{94}$$

$$y_k = \begin{bmatrix} C & -CT \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ \nu_k \end{bmatrix}, \tag{95}$$

and the physical rejection of faults will be obtained if and only if T and G satisfy the algebraic equations

$$(I-A)T + F = -BG, \tag{96}$$

$$CT = 0. \tag{97}$$

Under the existence condition for a solution to (96) and (97), i.e.,

$$\text{rang} \begin{bmatrix} A-I & B & -F \\ C & 0 & 0 \end{bmatrix} = \text{rang} \begin{bmatrix} I-A & B \\ C & 0 \end{bmatrix}, \tag{98}$$

for the gain G of the control law (91), the solution of (96) yields

$$G = [C(I-A)^{-1}B]^{-1}C(I-A)^{-1}F \tag{99}$$

and $T = (I-A)^{-1}(BG - F)$. Under (99), (93) gives

$$\begin{bmatrix} \bar{x}_{k+1} \\ \nu_{k+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ \nu_k \end{bmatrix} \tag{100}$$

$$+ \begin{bmatrix} B \\ 0 \end{bmatrix} u_k^n + \begin{bmatrix} I \\ 0 \end{bmatrix} w_k, \tag{101}$$

$$y_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ \nu_k \end{bmatrix} + v_k, \tag{102}$$

where $\bar{x}_k$ represents the state of the jump-free system. Thus, under (A, B) controllable, the LQG regulator $u_k^n = -\bar{L}\hat{\bar{x}}_k$ can be designed on the jump-free system h_0 (from the separation principle) to obtain the nominal system performances (not defined here). The reconfigurated control law, which reject q fault uncontrollable modes, is given by

$$u_k = -\bar{L}\hat{\bar{x}}_k - G\hat{\nu}_k \tag{103}$$

from the two-stage Kalman filter or, equivalently,

$$u_k = -\begin{bmatrix} \bar{L} & G \end{bmatrix} \begin{bmatrix} I & -\zeta_k \\ 0 & I \end{bmatrix} \begin{bmatrix} I & \zeta_k \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{\bar{x}}_k \\ \hat{\nu}_k \end{bmatrix}$$

$$= -\begin{bmatrix} \bar{L} & G - \bar{L}\zeta \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{\nu}_k \end{bmatrix}, \tag{104}$$

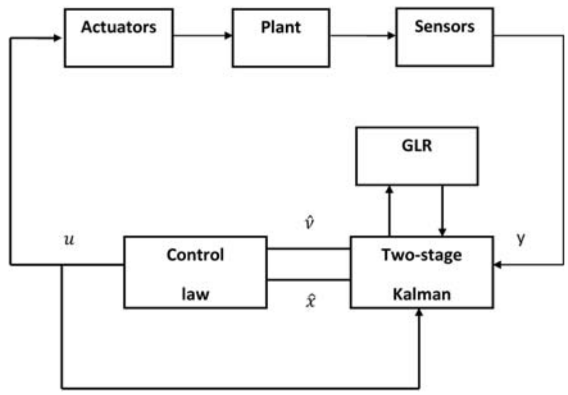

Fig. 1. Reconfigurable FTCS scheme based on the active GLR test.

from the augmented state Kalman filter. Note that, after each detection and isolation of one fault, the nominal control law $u_k^n = -\bar{L}\hat{\bar{x}}_k$ is not affected by the active GLR test but only corrected by the additive term $G\hat{\nu}_k$ depending on the old fault estimate (improved with the measurements available after their detection). The active GLR test depends only on the state prediction errors of the Kalman filter decoupled from u_k, and we can propose the reconfigurable FTCS scheme of Fig. 1.

The reference model used for the design of the control law GLR test coincides with that used by the GLR test. After each detection and isolation of one fault, the reference model is updated with the new state of the fault and the three parts of the FTCS, i.e., the GLR detector, the Kalman filter and the control law, can be reconfigured in harmony by the reconfiguration mechanism. To reduce the computational requirements, the passive GLR test working on a fixed reference model can be used but the statistical performances of the reconfigurable FTCS will be closely related by the rates of false alarms and good decisions of the statistical test used.

5. Results

To illustrate the proposed approach, we considered the system described by the following matrices:

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0.4 & 0.1 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{105}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \qquad F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \tag{106}$$

$$W = 0.01I, \qquad V = 0.5I, \tag{107}$$

where I is the identify matrix of the appropriate dimensions. Faults isolability is guaranteed with $\mathrm{rank}\,[CAf_1\ \ CAf_2]$ and $F = [f_1\ \ f_2]$. The statistical variables describing the performances of the reconfigurable FTCS coincide with those describing the performances of the statistical test. Thus, by simplifying the Monte Carlo simulation, the proposed example will be realized in open loop.

In the field of dynamic systems, the signal-to-noise ratio $\delta_i(k, r)$ is generally greater than the signal to noise ratio is in the fields of electrocardiogram analysis or geophysical signal processing, and the size M of the sliding window $W = [k - M \leq \tilde{r} \leq k]$ can be generally chosen small.

First, we suppose one fault occurred at 350 s with magnitude 2, and obtain the results of Fig. 2.

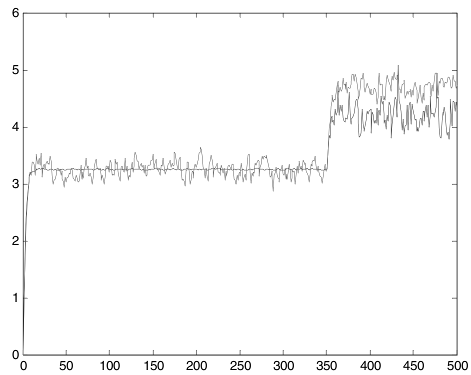

Fig. 2. First state component and its estimate produced by Willsky's algorithm.

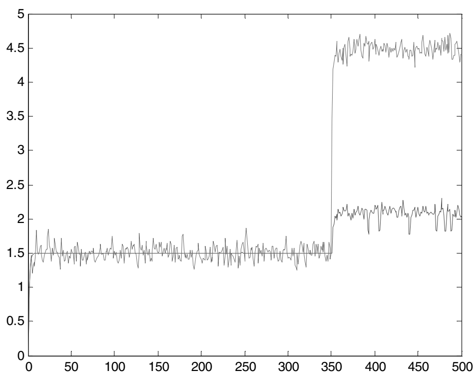

Fig. 3. Second state component and its estimate produced by Willsky's algorithm.

In the first case, the results show that the proposed state estimate given by our filter is more adaptive to the fault occurrence than Willsky's algorithm.

In the second case, we suppose that two sequential faults with magnitude 2, occurred at 350 s and 400 s. Figures 3–21 display the corresponding results.

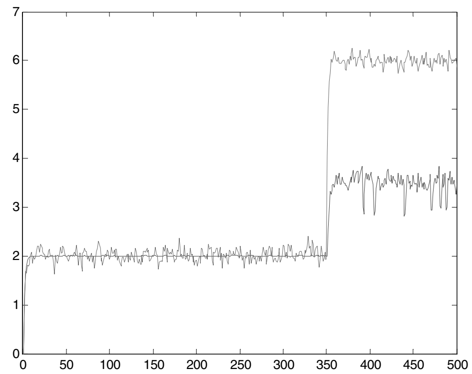

Fig. 4. Third state component and its estimate produced by Willsky's algorithm.

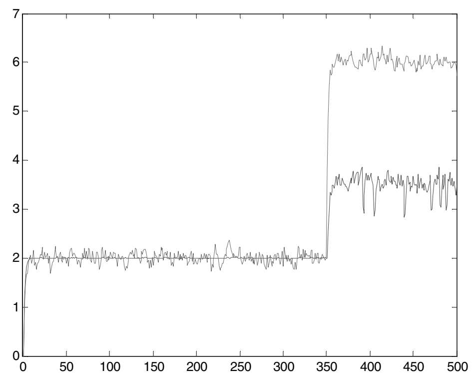

Fig. 5. Fourth state component and its estimate produced by Willsky's algorithm.

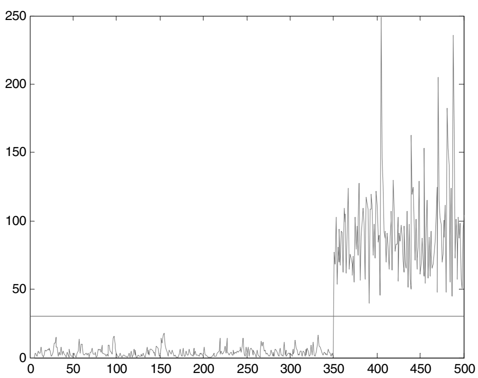

Fig. 6. GLR test applied to Willsky's algorithm.

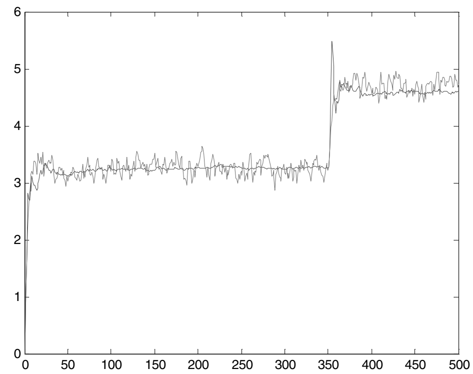

Fig. 7. First state component and its estimate produced by our adaptive filter algorithm.

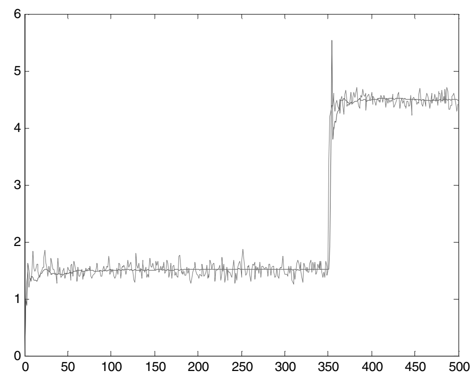

Fig. 8. Second state component and its estimate produced by our adaptive filter algorithm.

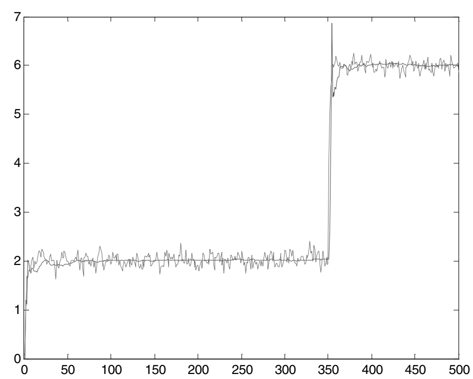

Fig. 9. Third state component and its estimate produced by our adaptive filter algorithm.

In the case of two sequential faults, the GLR detector applied to the augmented model allows detecting the first fault at 350 s and the second one at 400 s. Willsky's GLR detects just the first fault at 350 s, and cannot detect the second.

Remark 1. We also computed the rate of false alarms and the rate of good detections with 10^5 Monte Carlo trials. We obtained $\hat{P}^F \simeq 0.01$, $\hat{P}_D \simeq 0.85$ for the modified GLR test, and $\hat{P}^F \simeq 0.0055$, $\hat{P}^D \simeq 0.91$ for the passive GLR test which is clearly much powerful. We conclude that the passive GLR test is very powerful when quick detections lead to bad fault estimates and thus very useful for FTCS to maximize the rate of good decisions specially in regard with the occurrence of a considerable fault which may greatly affect the nominal system performance.

Remark 2. We can improve this approach of detection and isolation with an active GLR based on a free model

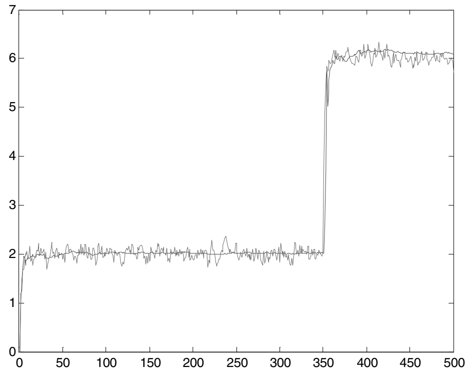

Fig. 10. Fourth state component and its estimate produced by our adaptive filter algorithm.

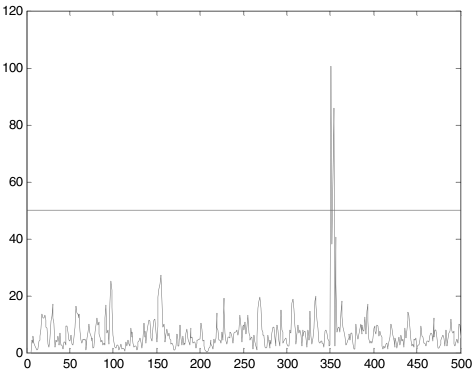

Fig. 11. GLR test applied to our adaptive filter.

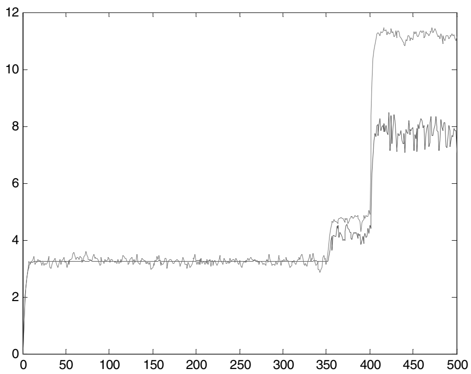

Fig. 12. First state component and its estimate produced by Willsky's algorithm in the presence of two sequential faults.

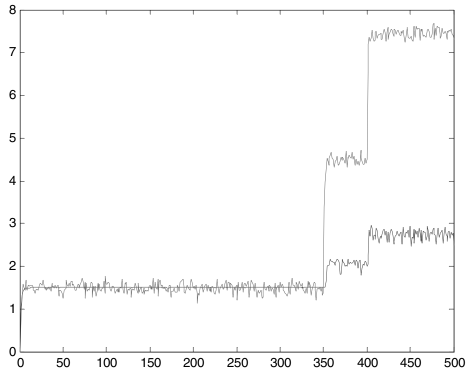

Fig. 13. Second state component and its estimate produced by Willsky's algorithm in the presence of two sequential faults.

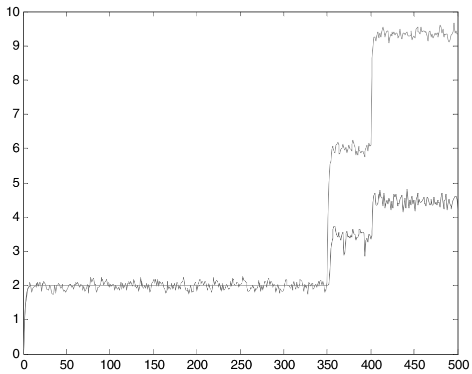

Fig. 14. Third state component and its estimate produced by Willsky's algorithm in the presence of two sequential faults.

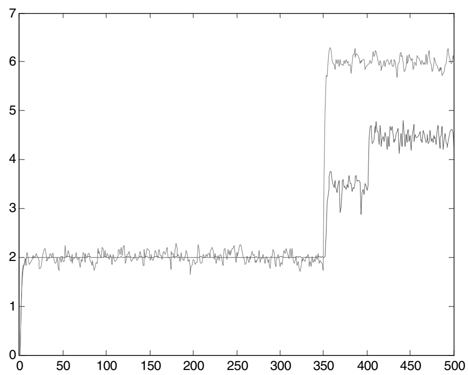

Fig. 15. Fourth state component and its estimate produced by Willsky's algorithm in the presence of two sequential faults.

which will be augmented after each detection and isolation. The faults already detected and isolated will be considered as perturbations and we will update the new GLR in order to detect another fault.

Sequential multiple decision theory is not complete and the choice of the threshold level ε is not studied in this paper. However, some simulation results not presented in this paper show that only the statistical tuning parameter ε can be fixed at the beginning of the processing (it is not the threshold level which is adaptive, but the augmented Kalman filter).

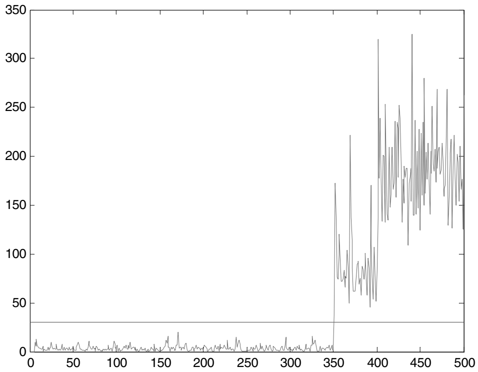

Fig. 16. GLR test applied to Willsky's algorithm in the presence of two sequential faults.

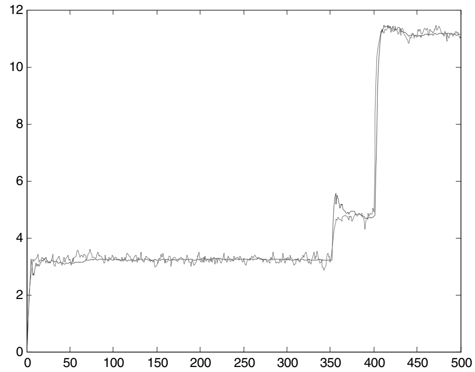

Fig. 17. First state component and its estimate produced by our adaptive algorithm in the presence of two sequential faults.

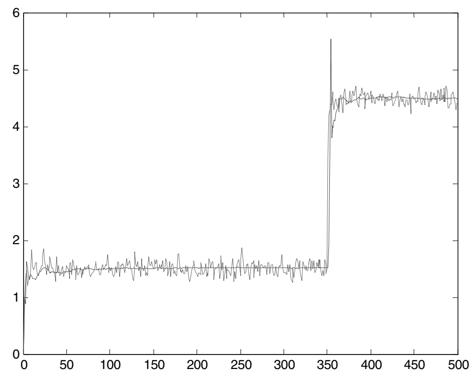

Fig. 18. Second state component and its estimate produced by our adaptive algorithm in the presence of two sequential faults.

6. Conclusion

Derived from the works of Willsky and Jones (1976), this paper has presented an active GLR test for sequential fault detection in stochastic discrete-time linear systems. From a new updating strategy based on the statistical rejection of the faults detected and isolated during the recursive

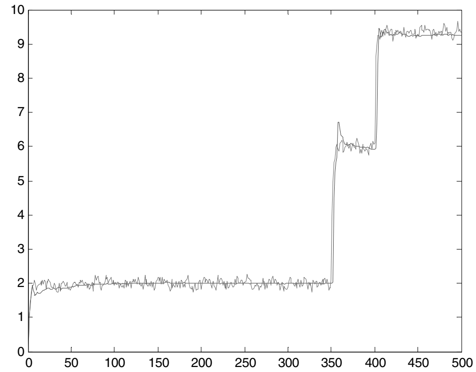

Fig. 19. Third state component and its estimate produced by our adaptive algorithm in the presence of two sequential faults.

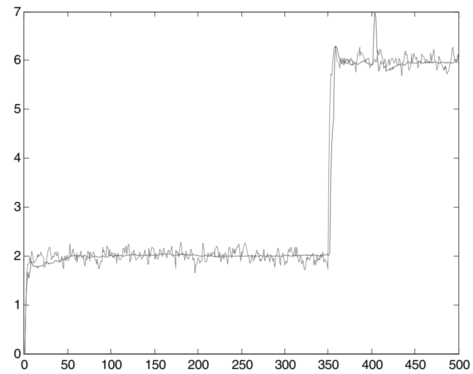

Fig. 20. Fourth state component and its estimate produced by our adaptive algorithm in the presence of two sequential faults.

treatment, the rate of false alarms was minimized and the rate of good decisions maximized. The active GLR test was integrated in a reconfigurable fault-tolerant control system by using an LQG regulator designed for the jump-free system where the nominal control law is corrected on-line to asymptotically reject the influence of faults.

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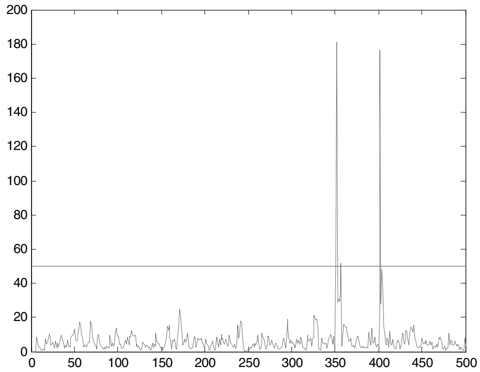

Fig. 21. GLR test applied to the adaptive algorithm in the presence of two sequential faults.

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Appendix

The predictive form of Friendland's two-stage Kalman filter optimally implements the following augmented state Kalman filter:

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{\nu}_{k+1} \end{bmatrix} = \hat{X}_{k+1} = \bar{A}\hat{X}_k + \bar{B}u_k + K_k\gamma_k, \quad (108)$$

$$\begin{bmatrix} P^x_{k+1} & P^{x\nu}_{k+1} \\ P^{\nu x}_{k+1} & P^{\nu}_{k+1} \end{bmatrix} = \Omega_{k+1} \quad (109)$$

$$\begin{aligned} &= \bar{A}\Omega_k\bar{A}^T + \bar{\Gamma}W\bar{\Gamma}^T \\ &\quad - \bar{A}\Omega_k\bar{C}^T\left(\bar{C}\Omega_k\bar{C}^T + V\right)^{-1}\bar{C}\Omega_k\bar{A}^T, \end{aligned} \quad (110)$$

$$K_k = \begin{bmatrix} K_k^x \\ K_k^{\nu^j} \end{bmatrix} = \bar{A}\Omega_k \bar{C}^T H_k^{-1}, \quad H_k = \bar{C}\Omega\bar{C}^T + V, \tag{111}$$

with

$$\hat{X}_0 = \begin{bmatrix} \hat{x}_0 \\ \hat{\nu}_0 \end{bmatrix}, \quad \Omega_0 = \begin{bmatrix} P_0^x & P_0^{x\nu} \\ P_0^{\nu x} & P_0^{\nu} \end{bmatrix},$$

where

$$\bar{A} = \begin{bmatrix} A & F \\ 0 & I \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$
$$\bar{\Gamma} = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$\hat{x}_{k+1} = \hat{\bar{x}}_{k+1} + \zeta_{k+1}\hat{\nu}_{k+1}, \tag{112}$$
$$P_{k+1} = \bar{P}_{k+1} + \zeta_{k+1} P_{k+1}^{\nu} \zeta_{k+1}^T. \tag{113}$$

Here $(\hat{\bar{x}}_{k+1}, \bar{P}_{k+1})$ are given by the bias-free filter

$$\hat{\bar{x}}_{k+1} = A\hat{\bar{x}}_k + Bu_k + \bar{K}_k(y_k - C\hat{\bar{x}}_k), \tag{114}$$
$$\bar{P}_{k+1} = A\bar{P}_k A^T + W - A\bar{P}_k C^T (\bar{H}_k)^{-1} C\bar{P}_k A^T, \tag{115}$$
$$\bar{K}_k = A\bar{P}_k C^T \bar{H}_k^{-1}, \tag{116}$$
$$\bar{H}_k = C\bar{P}_k C^T + V, \tag{117}$$

where $(\hat{\nu}_{k+1}, P_{k+1}^{\nu})$ are given by the bias filter

$$\hat{\nu}_{k+1} = \hat{\nu}_k + K_k^{\nu}\gamma_k, \tag{118}$$
$$P_{k+1}^{\nu} = P_k^{\nu} - P_k^{\nu}\varrho_k^T H_k^{-1}\varrho_k P_k^{\nu}, \tag{119}$$
$$K_k^{\nu} = P_k^{\nu}\varrho_k^T H_k^{-1}, \tag{120}$$
$$\gamma_k = \bar{\gamma}_k - \varrho_k\hat{\nu}_k, \tag{121}$$
$$H_k = \bar{H}_k + \varrho_k P_k^{\nu}\varrho_k^T. \tag{122}$$

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